Engineering of musculoskeletal system and rehabilitation

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Musculoskeletal conditions

<u>WHO</u> (February 2021):

- Approximately 1.71 billion people have musculoskeletal conditions worldwide.
- Musculoskeletal conditions are the leading contributor to disability worldwide

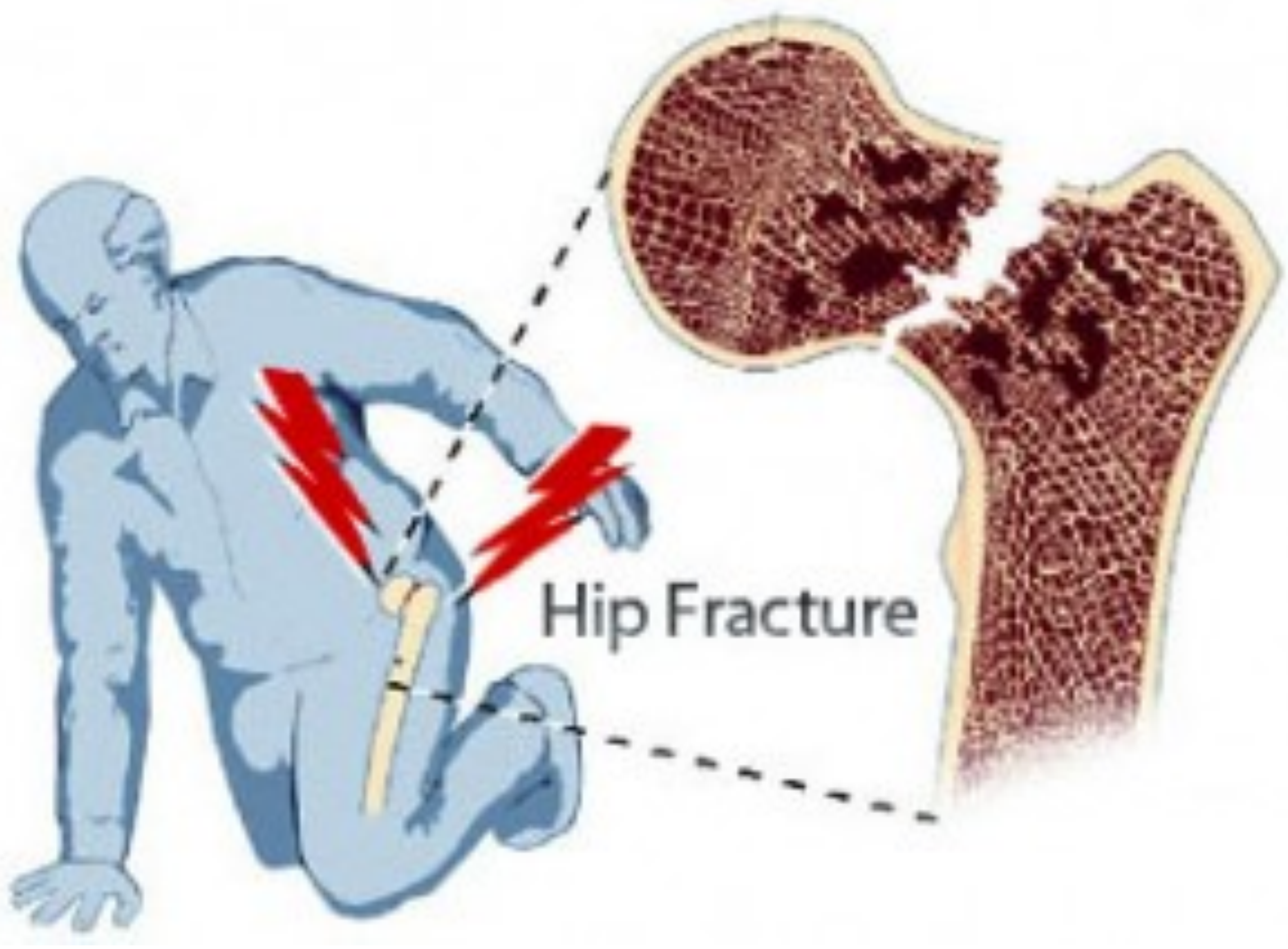
<u>Musculoskeletal conditions include:</u>

- Osteoarthritis
- Osteoporosis, osteopenia and associated fragility fractures, traumatic fractures of soft and hard tissues

All age groups of the population are affected

Biomechanical aspects are involved in most musculoskeletal conditions

250’000 ACL injuries occur in the USA annually.



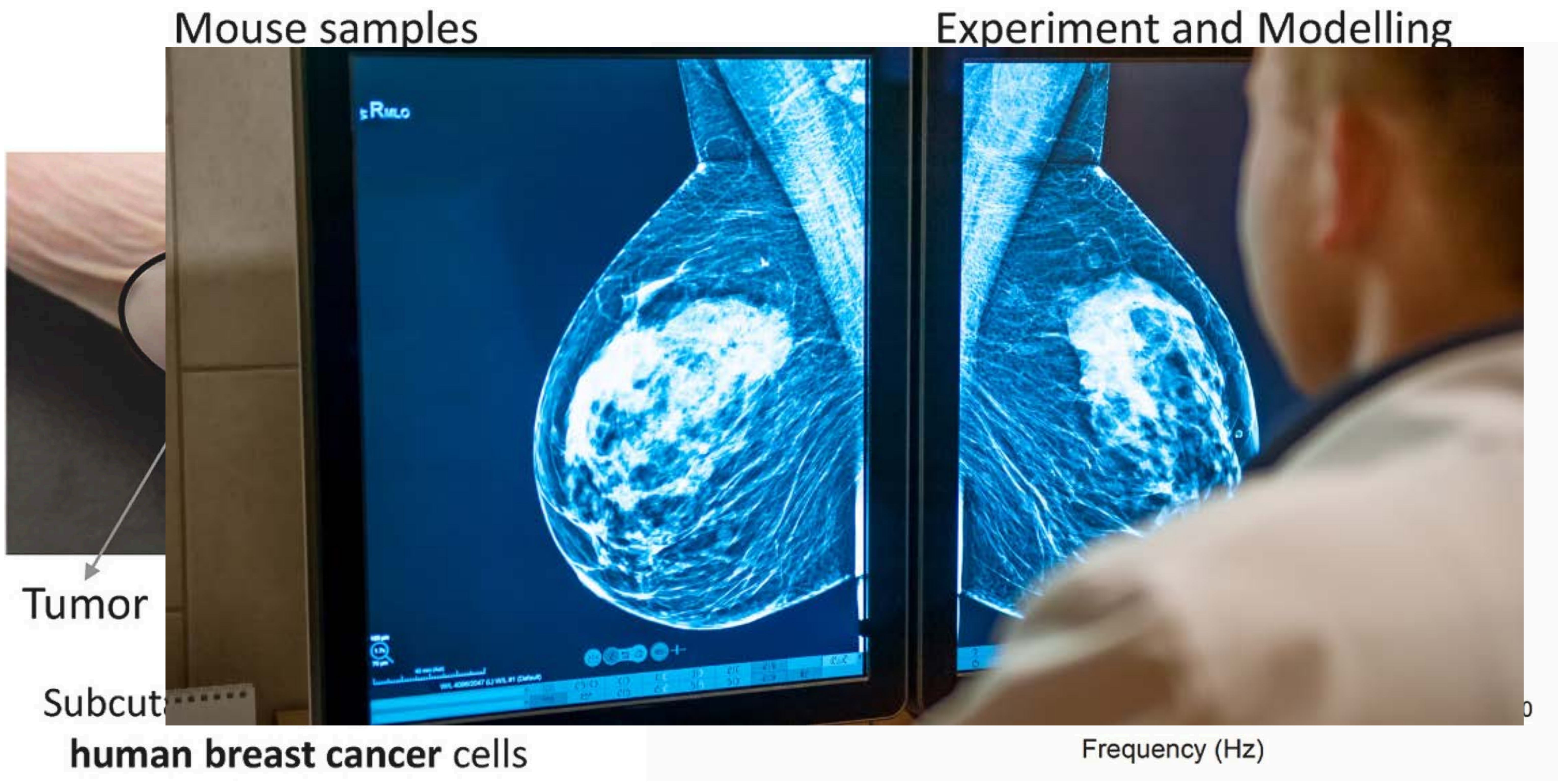
Mortality rate is 20% within 1 year.

How can mechanical aspects be considered in medicine?

1) Diagnostic

Elastography uses wave propagation to evaluate the mechanical properties of the tissues

The shear modulus of tumor tissues may be an order of magnitude higher than in normal tissues.



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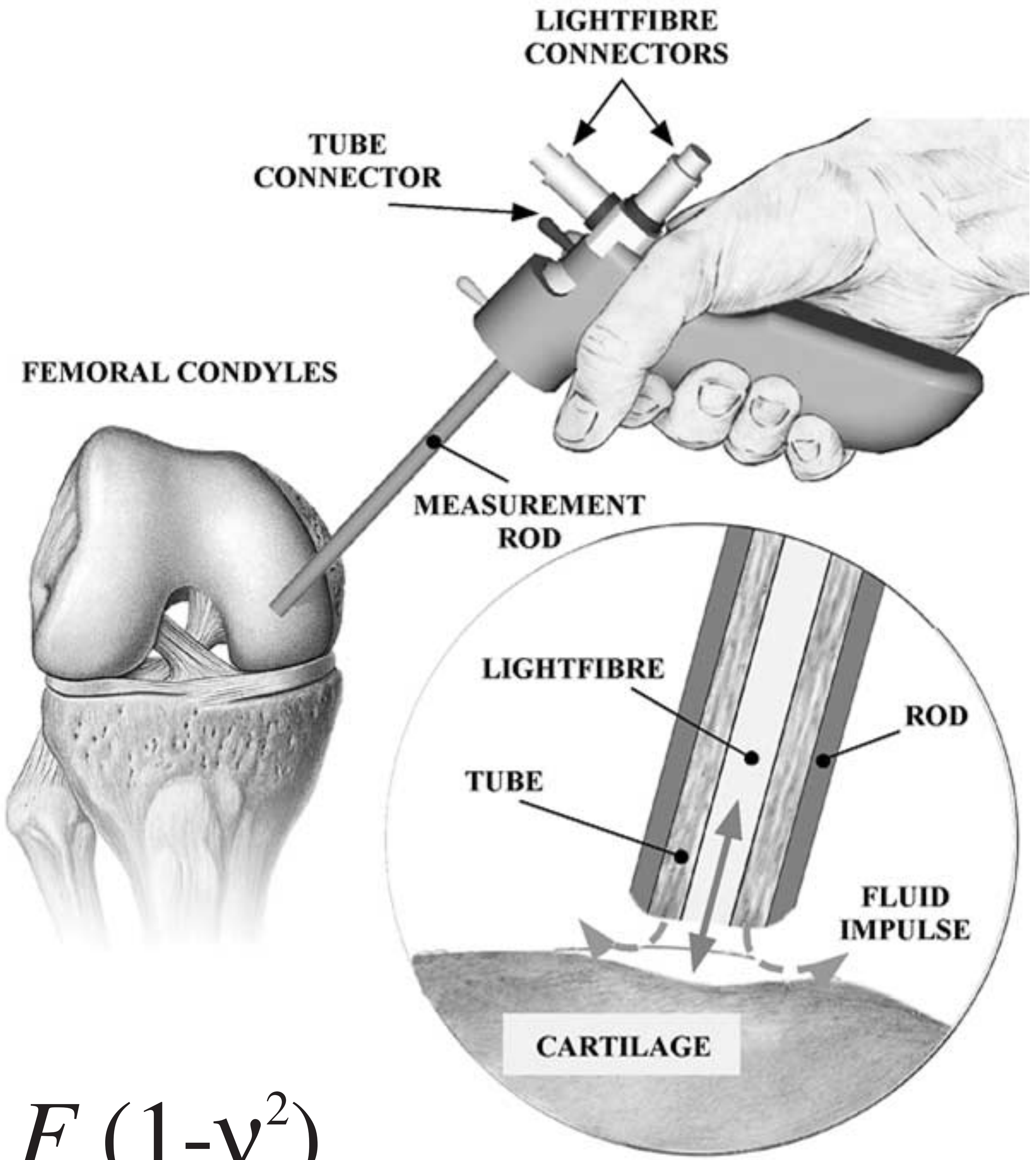
How can mechanical aspects be considered in medicine?

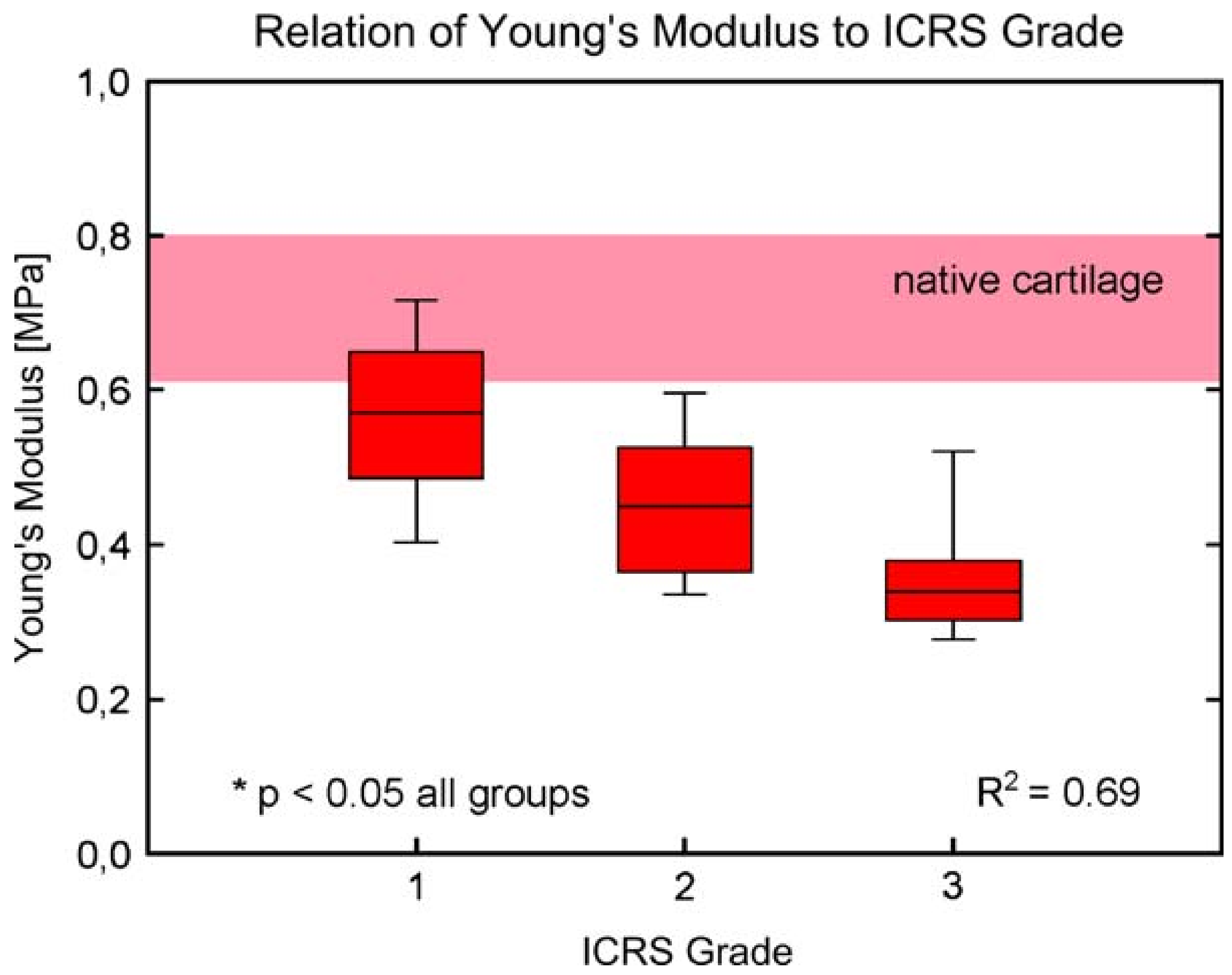
1) Diagnostic

The cartilage aging process weakens its mechanical properties



$$E = \frac{F\,(1-\nu^2)}{2a\omega\kappa}$$

10.1177/0363546503261725



ICRS grading based on the Outerbridge score[6]

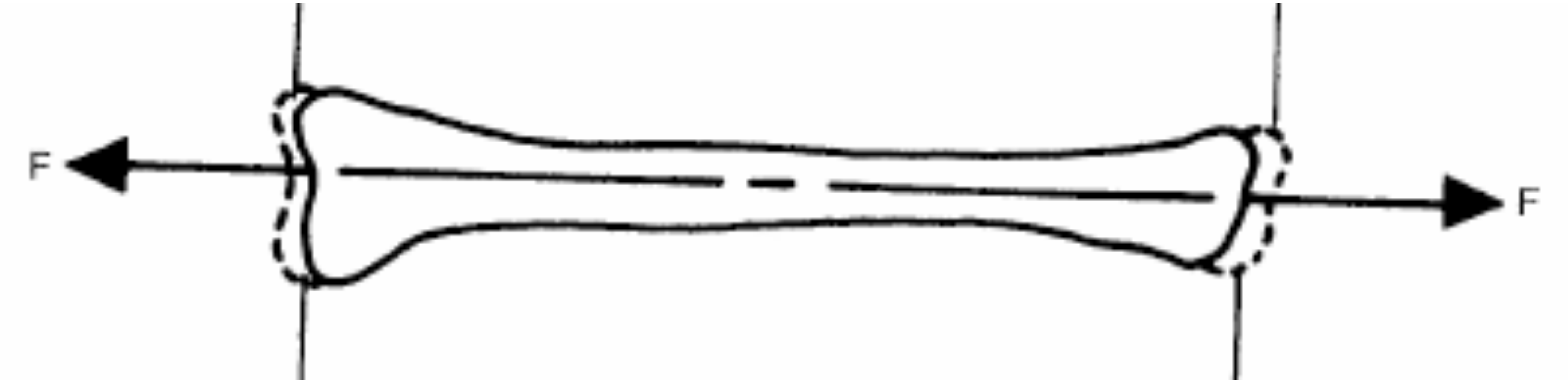
Grade	Property
1	Superficial lesions, fissures and cracks, soft indentation
2	Fraying, lesions extending down to <50% of cartilage depth
3	Partial loss of cartilage thickness, cartilage defects extending down >50% of cartilage depth as well as down to calcified layer
4	Complete loss of cartilage thickness, bone only

How can mechanical aspects be considered in medicine?

2) Therapeutics

- prevention of fracture (car accident, sport traumatology, …)
- implant design (orthopedic, cardiac valves, …)
- tissue differentiation
- physical medicine
- prevention of disease (osteoporosis)
- drug delivery
- muscle biomechanics
- brain model (damage)
- tissue engineering
  - stiffness of matrice affects cell behavior
  - functional tissue engineering + permeability

Biological tissues are stressed and deformed when a force or moment is applied



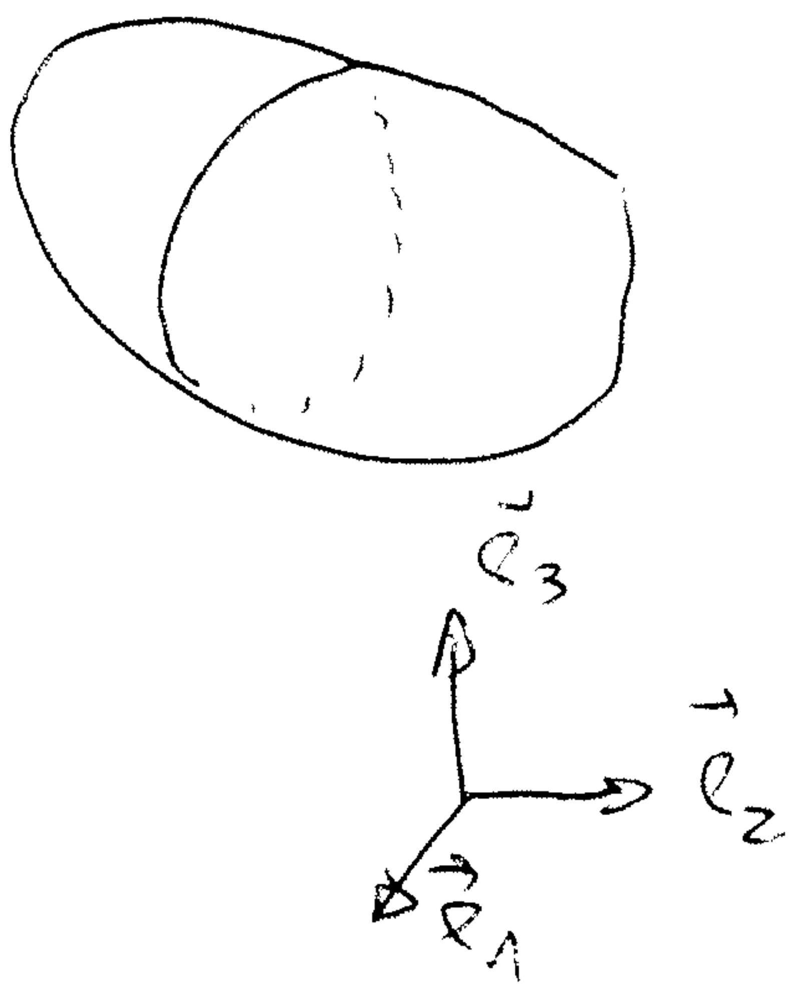
Newton equations of motion $m\vec{a} = \sum \vec{F}^{ext}$ $\quad I\dot{\omega} = \sum \overrightarrow{M}_o^{ext}$

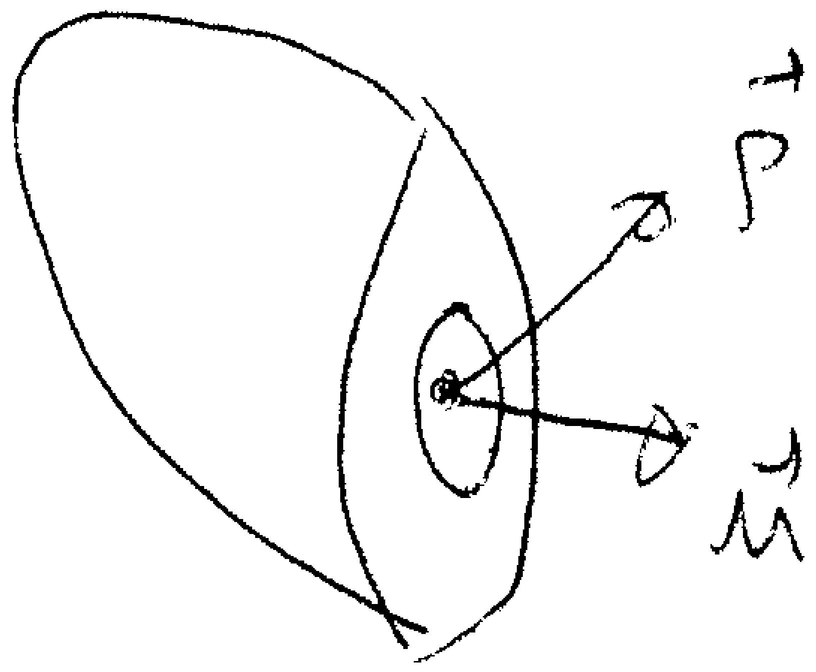
Continuum mechanics concepts should be considered

Basic concepts of continuum mechanics

- Conservation laws
  - linear momentum
  - angular momentum
  - mass
  - energy
  - entropy inequality

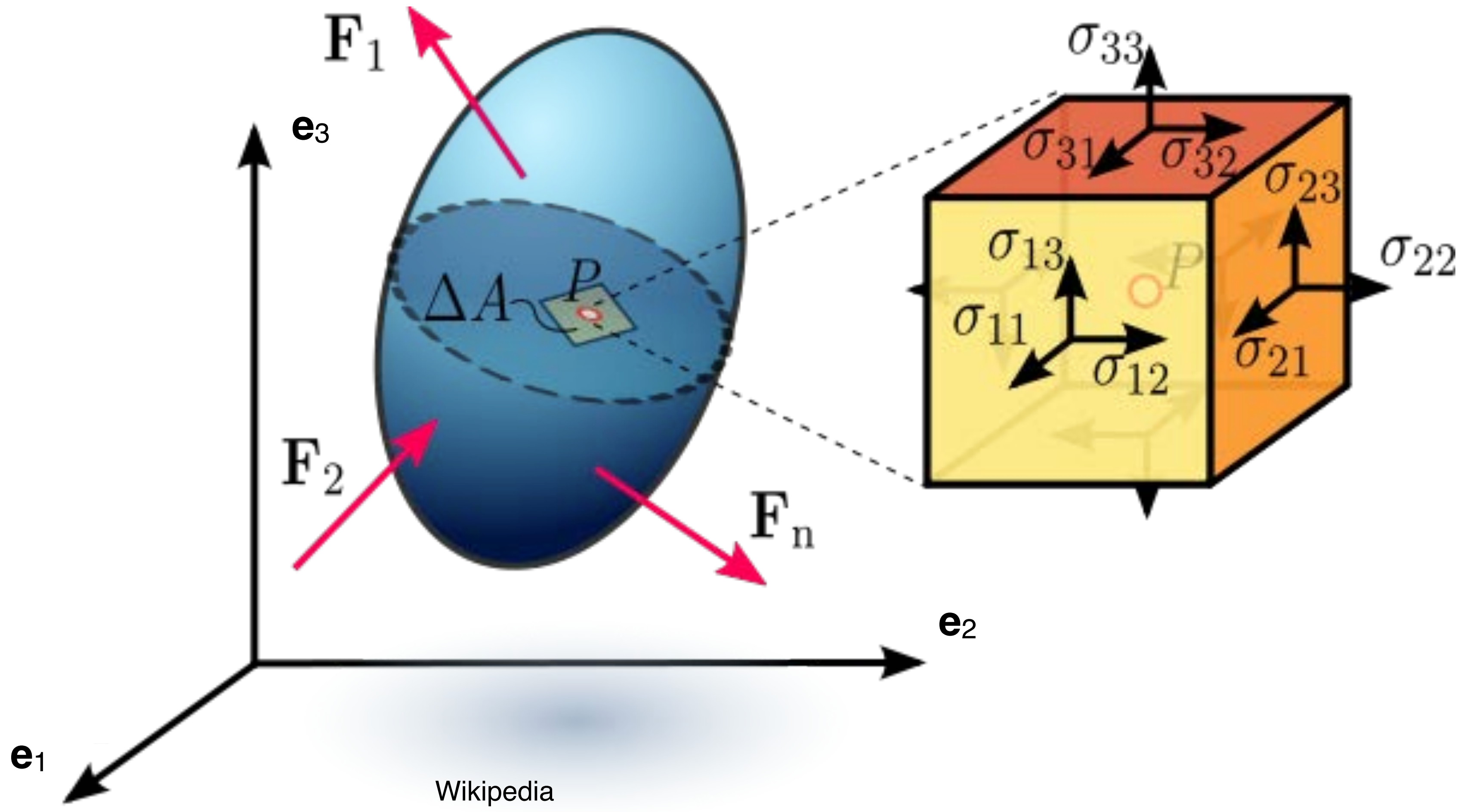
The mathematical "*nature*" of a stress and a strain is a second order tensor





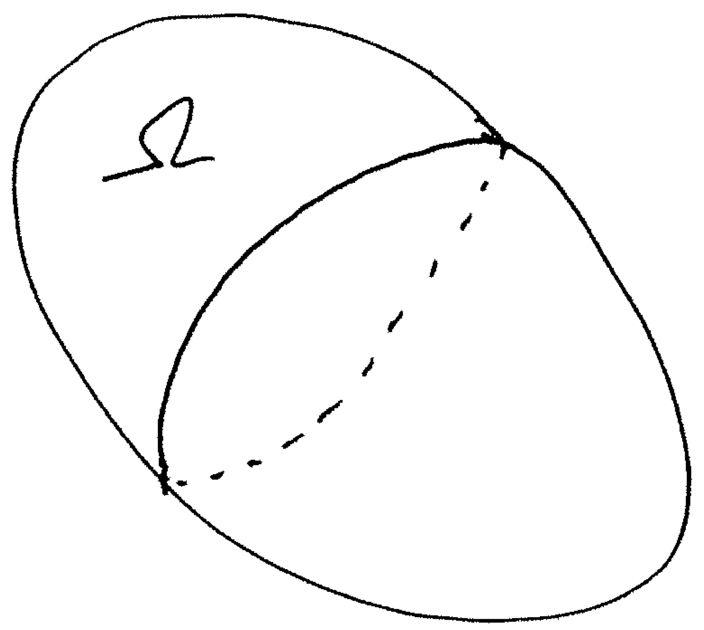
$$\boldsymbol{p} = \boldsymbol{\sigma n}$$

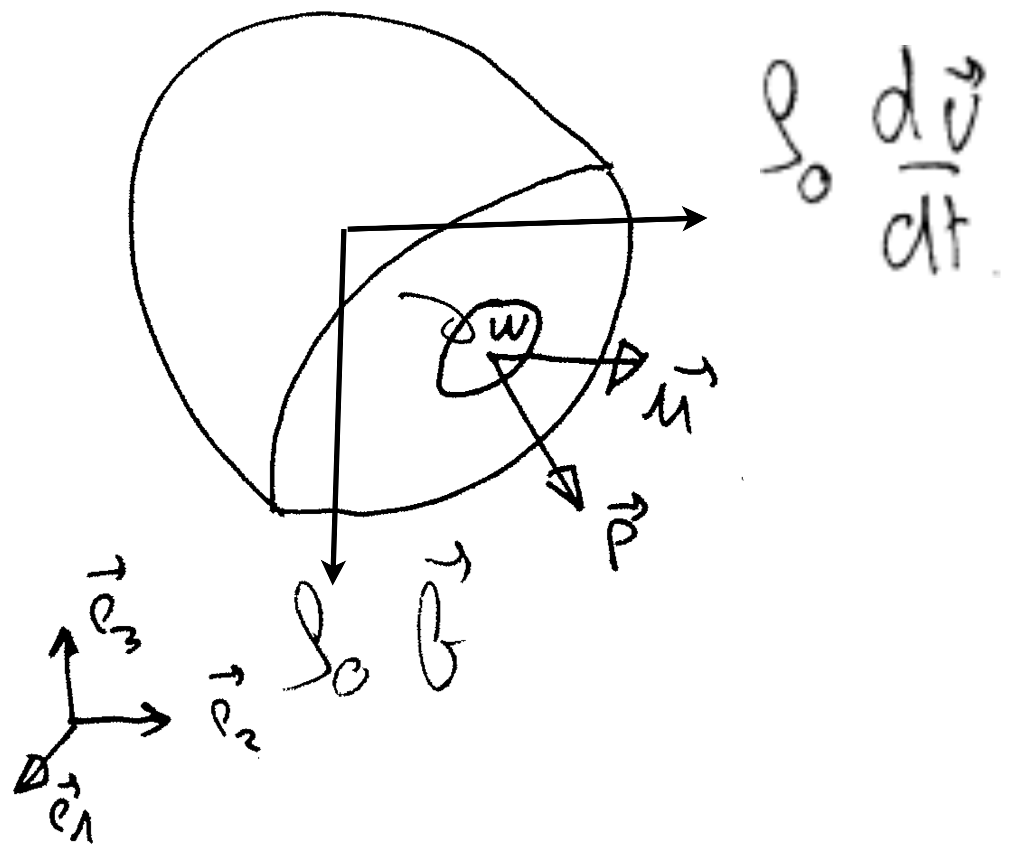
$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$



σ_{ij} : stress tensor components

Conservation of the linear momentum





$$\frac{d}{dt}\int_{\omega} \rho_0 \vec{v} \, dV = \int_{\partial\omega} \vec{p} \, dA + \int_{\omega} \rho_0 \vec{b} \, dV \qquad \forall \ \omega \in \Omega$$

Conservation of the linear momentum (surface to volume integration)

$$\frac{d}{dt}\int_{\omega} \rho_0 \vec{v}\, dV = \int_{\partial\omega} \sigma\, \vec{n}\, dA + \int_{\omega} \rho_0 \vec{b}\, dV \qquad \forall\, \omega \in \Omega$$

Divergence theorem: $\displaystyle\int_{\partial\omega} \sigma\, \vec{n}\, dA = \int_{\omega} \operatorname{div}\sigma\, dV$

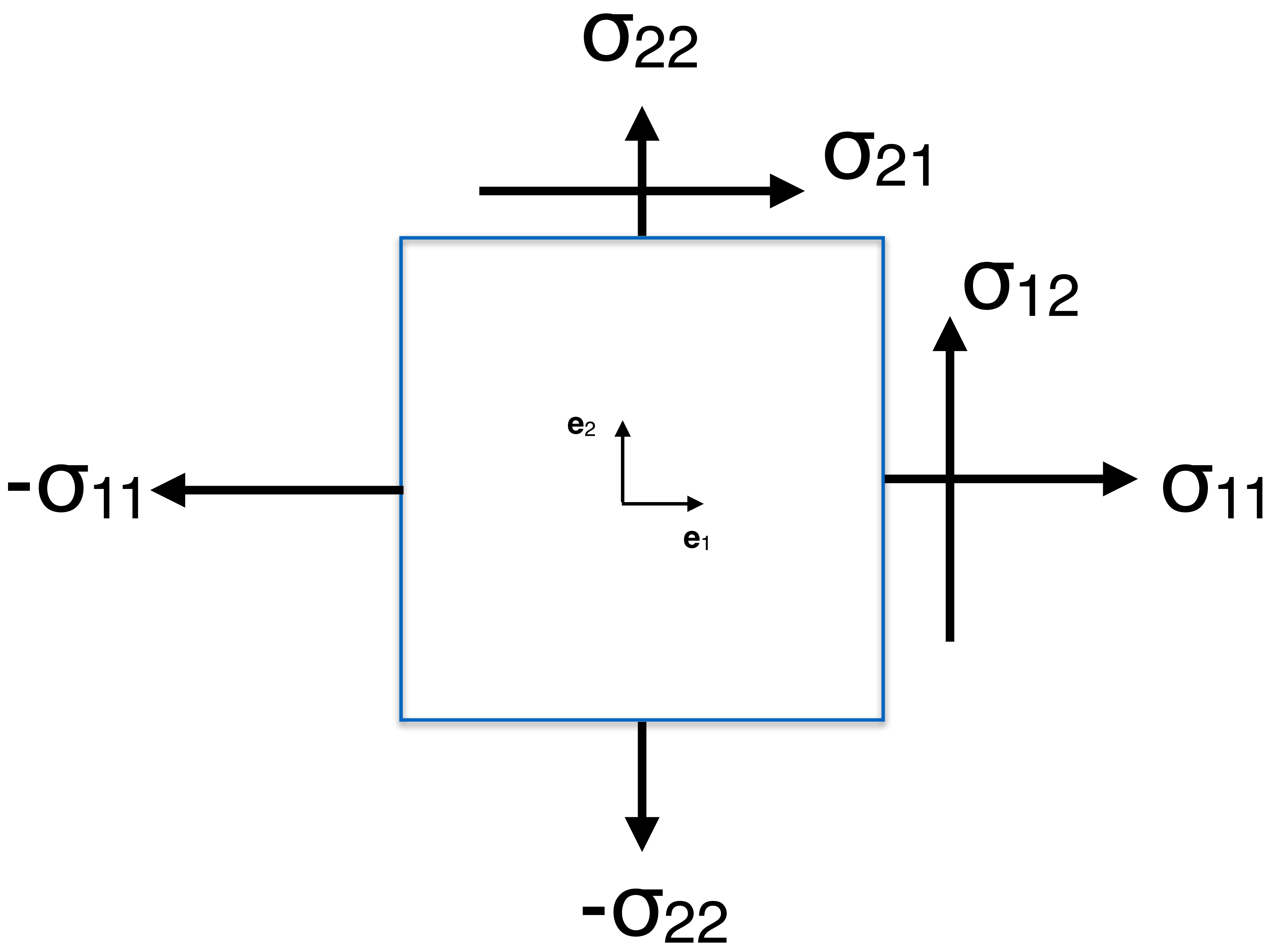
$$\boxed{\Rightarrow \frac{d}{dt}\int_{\omega} \rho_0 \vec{v}\, dV = \int_{\omega} \operatorname{div}\sigma\, dV + \int_{\omega} \rho_0 \vec{b}\, dV \qquad \forall\, \omega \in \Omega}$$

Conservation of the linear momentum (localisation)

$$\Rightarrow \frac{d}{dt}\int_{\omega} \rho_0 \vec{v}\, dV = \int_{\omega} \operatorname{div} \sigma \, dV + \int_{\omega} \rho_0 \vec{b}\, dV \qquad \forall\, \omega \in \Omega$$

$$\boxed{\rho_0 \frac{d\vec{v}}{dt} = \operatorname{div} \sigma + \rho_0 \vec{b}}$$

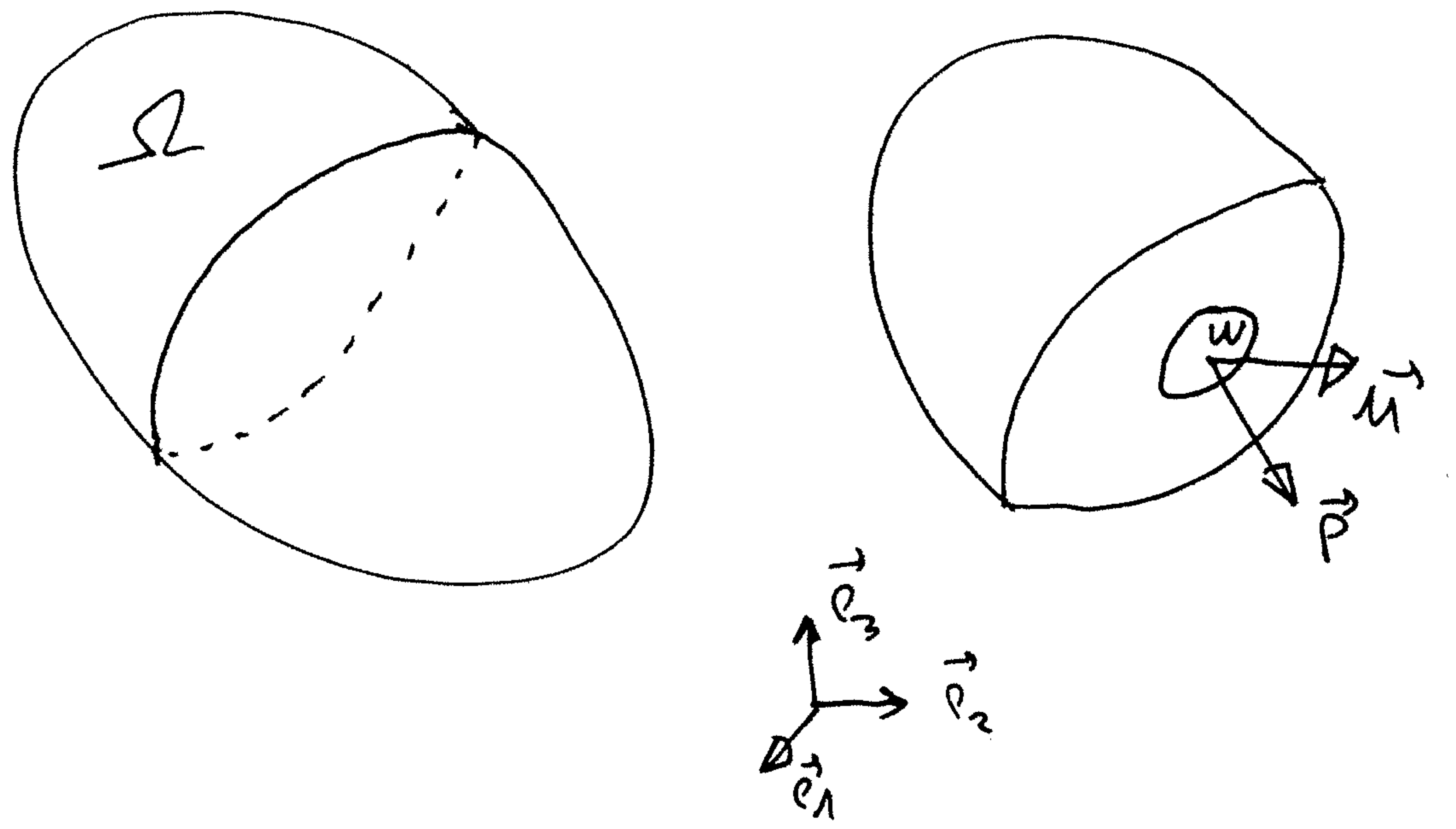
Conservation of the angular momentum: graphical interpretation



The satisfaction of the angular conservation momentum imposes:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$$

Conservation of the angular momentum



$$\frac{d}{dt}\int_{\omega} \vec{x} \wedge \rho_0 \vec{v}\, dV = \int_{\partial\omega} \vec{x} \wedge \vec{p}\, dA + \int_{\omega} \vec{x} \wedge \rho_0 \vec{b}\, dV \quad \forall \omega \in \Omega$$

Conservation of the angular momentum (math development)

$$\boxed{\frac{d}{dt}\int_{\omega} \rho_0\, \vec{x} \wedge \vec{v}\, dV = \int_{\partial\omega} \vec{x} \wedge (\sigma\, \vec{n})\, dA + \int_{\omega} \rho_0\, \vec{x} \wedge \vec{b}\, dV \quad \forall \omega \in \Omega}$$

$$\Rightarrow i)\quad \frac{d}{dt}\int_{\omega} \rho_0\, \vec{x} \wedge \vec{v}\, dV = \rho_0 \int_{\omega} \frac{d\vec{x}}{dt} \wedge \vec{v}\, dV + \rho_0 \int_{\omega} \vec{x} \wedge \frac{d\vec{v}}{dt}\, dV$$

$$(\rightarrow \vec{v} \wedge \vec{v} = 0)$$

$$\text{"="}\ \rho_0 \int_{\omega} \varepsilon_{ijk}\, x_j\, v_k\, dV$$

$$\varepsilon_{ijk} = \begin{cases} 1 \text{ if } (i,j,k) \text{ is an even permutation} \\ \quad (\varepsilon_{123} = \varepsilon_{312} = \varepsilon_{231} = 1) \\ -1 \text{ if } (i,j,k) \text{ is an odd permutation} \\ \quad (\varepsilon_{321} = \varepsilon_{132} = \varepsilon_{213} = -1) \\ 0 \text{ if any } (i,j,k) \text{ are equal} \end{cases}$$

$$\varepsilon_{112} = \varepsilon_{222} = \dots = 0$$

Conservation of the angular momentum (math development)

$$\frac{d}{dt}\int_{\omega} \rho_0\, \vec{x} \wedge \vec{v}\, dV = \int_{\partial\omega} \vec{x} \wedge (\sigma\, \vec{n})\, dA + \int_{\omega} \rho_0\, \vec{x} \wedge \vec{b}\, dV \quad \forall \omega \in \Omega$$

$$(ii) \int_{\partial\omega} \varepsilon_{ijk}\, x_j\, \sigma_{km}\, n_m\, dA$$

$$= \int_{\omega} \varepsilon_{ijk} \left(x_j \sigma_{km} \right)_{,m} dV = \int_{\omega} \varepsilon_{ijk} \left(\sigma_{kj} + x_j\, \sigma_{km,m} \right) dV$$

Conservation of the angular momentum (math development)

$$\boxed{\frac{d}{dt}\int_{\omega} \rho_0\, \vec{x} \wedge \vec{v}\, dV = \int_{\partial\omega} \vec{x} \wedge (\sigma\, \vec{n})\, dA + \int_{\omega} \rho_0\, \vec{x} \wedge \vec{b}\, dV \quad \forall \omega \in \Omega}$$

$$\text{iii)} \int_{\omega} \rho_0\, \vec{x} \wedge \vec{b}\, dV \text{ "=" } \int_{\omega} \rho_0\, \varepsilon_{ijk}\, x_j\, b_k\, dV$$

$$\Rightarrow \int_{\omega} \left\{ \varepsilon_{ijk}\, x_j \left(\rho_0 \dot{v}_k - \sigma_{km,m} - \rho_0 b_k\right) - \varepsilon_{ijk}\, \sigma_{kj} \right\} dV = 0$$

Conservation of the angular momentum (math development)

$$\boxed{\int_{W} \left\{ \varepsilon_{ijk}\, x_j \left(\rho_0 \dot{v}_k - T_{km,m} - \rho_0 b_k \right) + \left(\varepsilon_{ijk}\, T_{kj} \right) \right\} dV = 0}$$

$\rho_0 \dot{v}_k - T_{km,m} - \rho_0 b_k = 0$ (conservation of the linear momentum)

$$\Rightarrow \boxed{\varepsilon_{ijk} T_{kj} = 0}$$

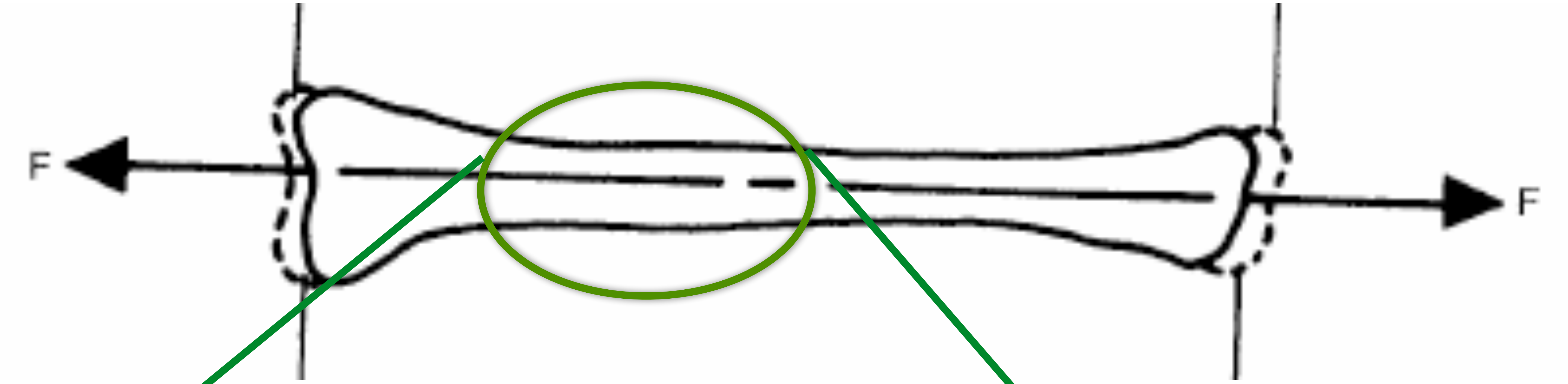
$i = 1 \quad \underbrace{\varepsilon_{123}}_{1} T_{32} + \underbrace{\varepsilon_{132}}_{-1} T_{23} = 0$

$T_{32} - T_{23} = 0 \quad \boxed{T_{32} = T_{23}}$

$i = 2 \ldots \quad T_{31} = T_{13}$

$i = 3 \ldots \quad T_{12} = T_{21}$

Biological tissues are stressed and deformed when a force or moment is applied



Newton equations of motion $m\vec{a} = \sum \vec{F}^{ext}$ $I\dot{\omega} = \sum \overrightarrow{M_o}^{ext}$

Cauchy momentum equation $\rho \frac{\partial v}{\partial t} = \nabla \cdot \sigma + \rho b$

$\boxed{\sigma = \sigma^T}$

One important aspect of biomechanics is then to characterise tissues through constitutive laws

$$\boxed{\rho \frac{d\boldsymbol{v}}{dt} = div\, \boldsymbol{\sigma} + \rho \boldsymbol{b}}$$

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}, \varepsilon_p, \ldots)$$

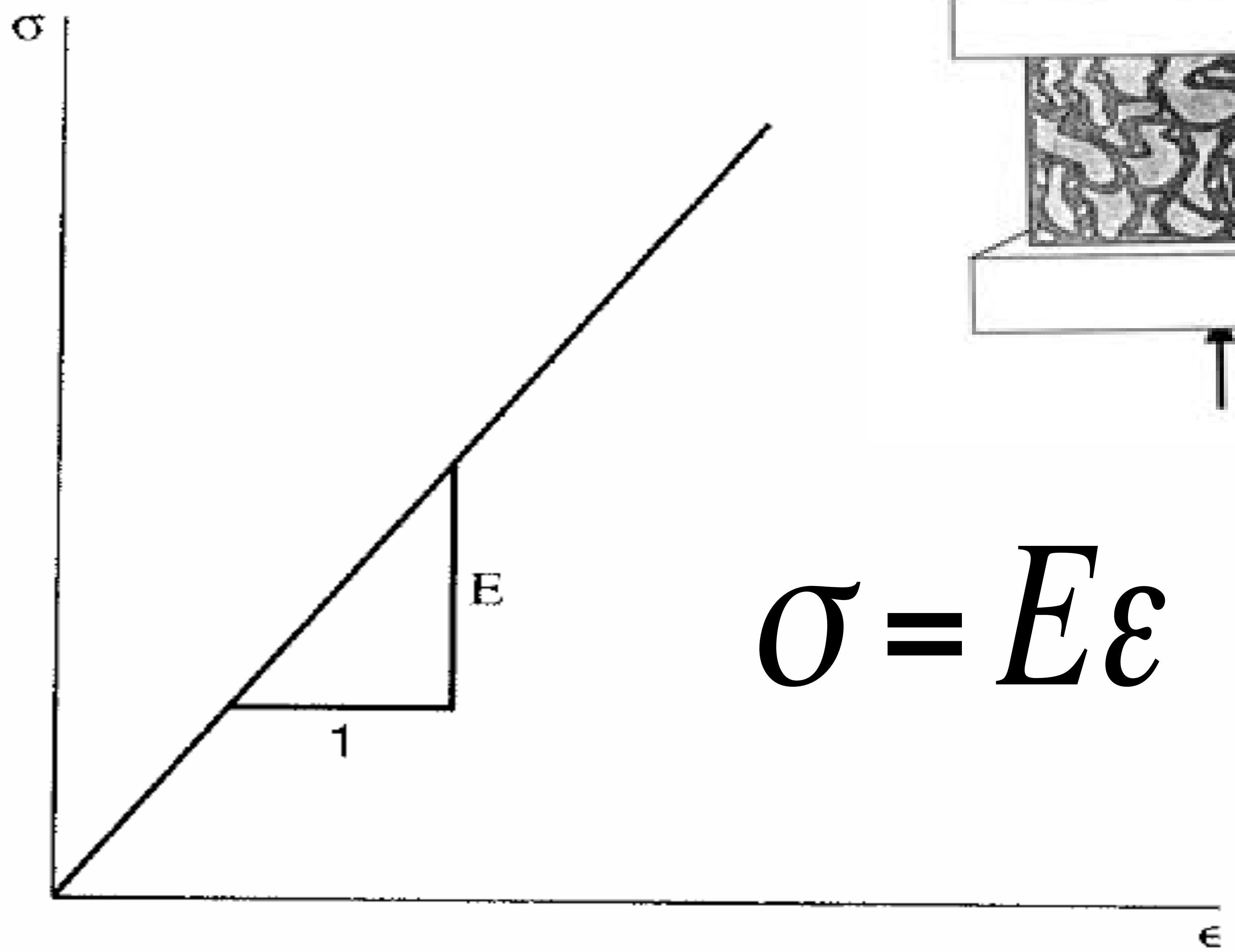
Elasticity -> $\sigma = \sigma(\varepsilon)$ → $\boxed{\text{Linear}}$ / Non-linear

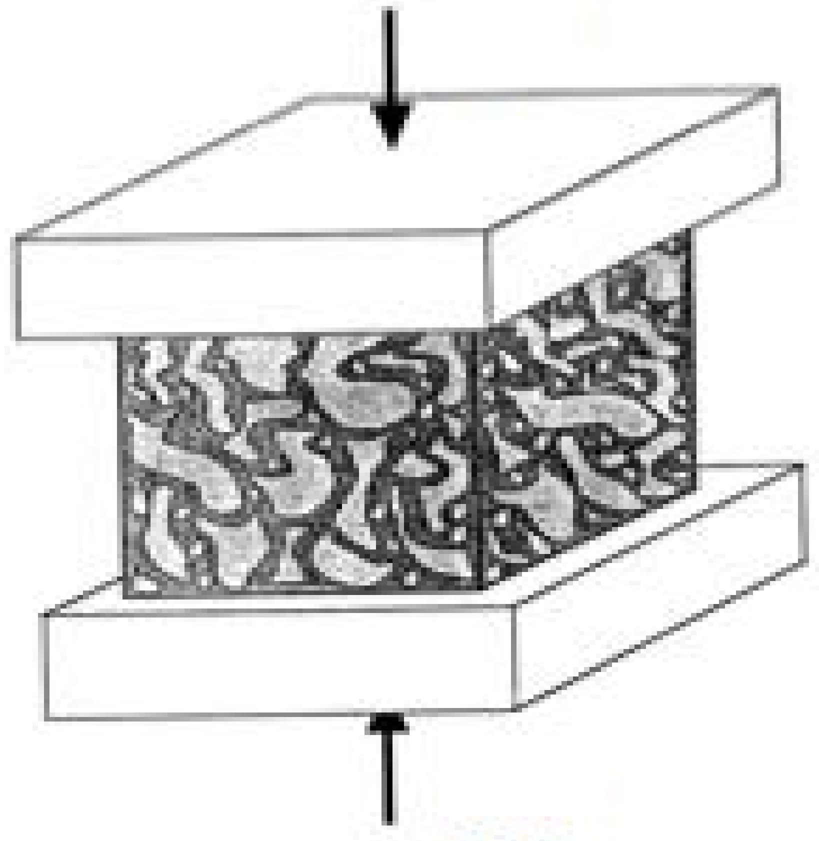
Biomechanics at the tissue level

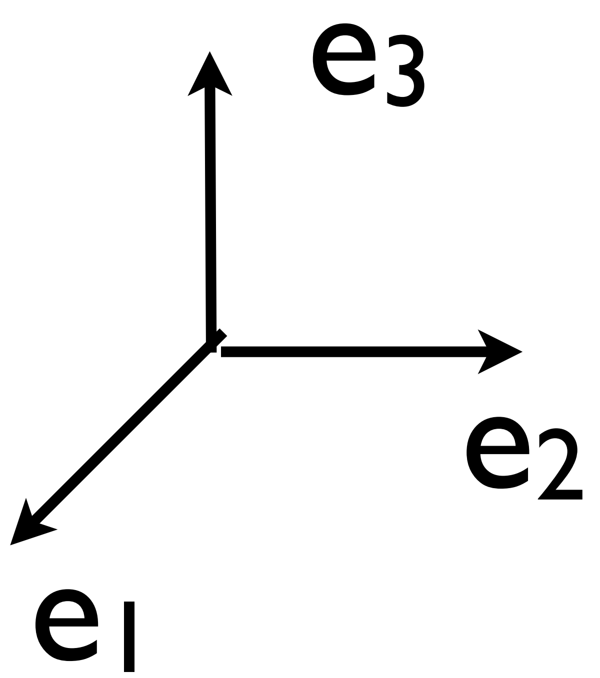
i) Continuum mechanics (conservation laws)

ii) Constitutive laws (linear, non-linear)

Hooke's Law in 1D







$$\sigma = E\varepsilon$$

$$\sigma_{33} = E\epsilon_{33}$$

E: Young's modulus

Hooke law in 3D (symmetries of the stiffness tensor C)

General linear relationship between the stress and the strain (**81** parameters)

$$\sigma_{ij} = C_{ijkl}\ \varepsilon_{kl}$$

Symmetry of the stress and corresponding strain tensors (**36** parameters)

$$C_{ijkl} \text{ -> } C_{\alpha\beta}$$

Matrix notation of Hooke's law (Voigt notation)

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} \equiv \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} \quad ; \quad [\boldsymbol{\epsilon}] = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \\ 2\epsilon_{12} \end{bmatrix} \equiv \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix}$$

$$[\boldsymbol{\sigma}] = [\boldsymbol{C}][\boldsymbol{\epsilon}]$$

$$[\boldsymbol{C}] = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1131} & C_{1112} \\ C_{2211} & C_{2222} & C_{2233} & C_{2223} & C_{2231} & C_{2212} \\ C_{3311} & C_{3322} & C_{3333} & C_{3323} & C_{3331} & C_{3312} \\ C_{2311} & C_{2322} & C_{2333} & C_{2323} & C_{2331} & C_{2312} \\ C_{3111} & C_{3122} & C_{3133} & C_{3123} & C_{3131} & C_{3112} \\ C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1231} & C_{1212} \end{bmatrix} := \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix}$$

Hooke law in 3D (symmetries of the stiffness tensor C)

Stress derived from a strain energy function *U* (**21** parameters)

$$\sigma_{ij} = \frac{\partial U}{\partial \epsilon_{ij}} \Longrightarrow C_{ijkl} = \frac{\partial^2 U}{\partial \epsilon_{ij} \partial \epsilon_{kl}}$$

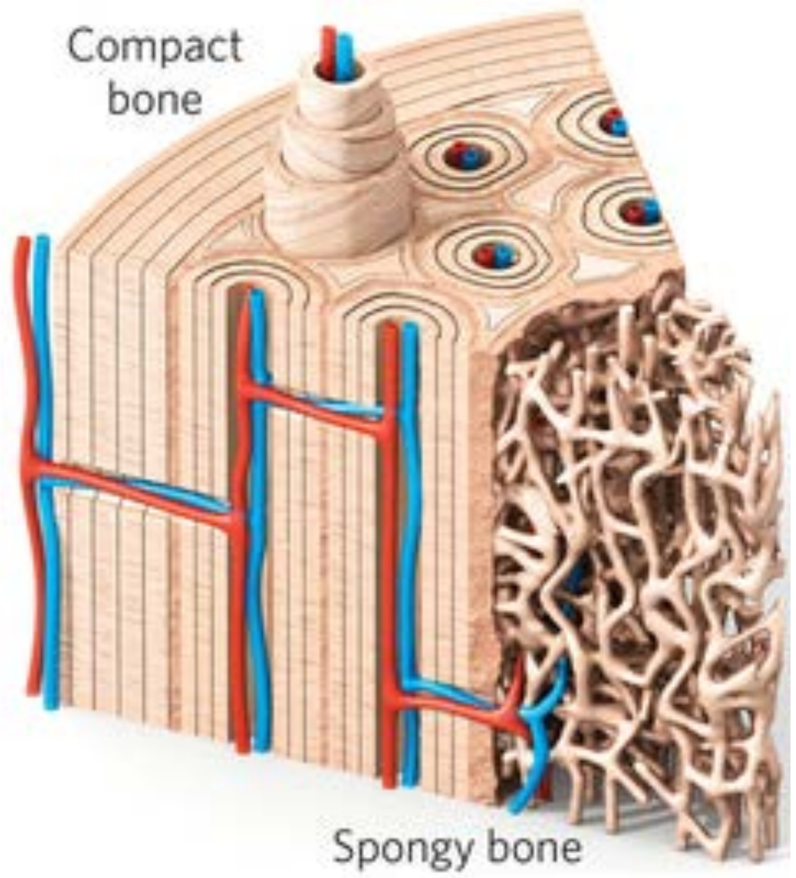
$C_{\alpha\beta}$ -> $C_{\beta\alpha}$

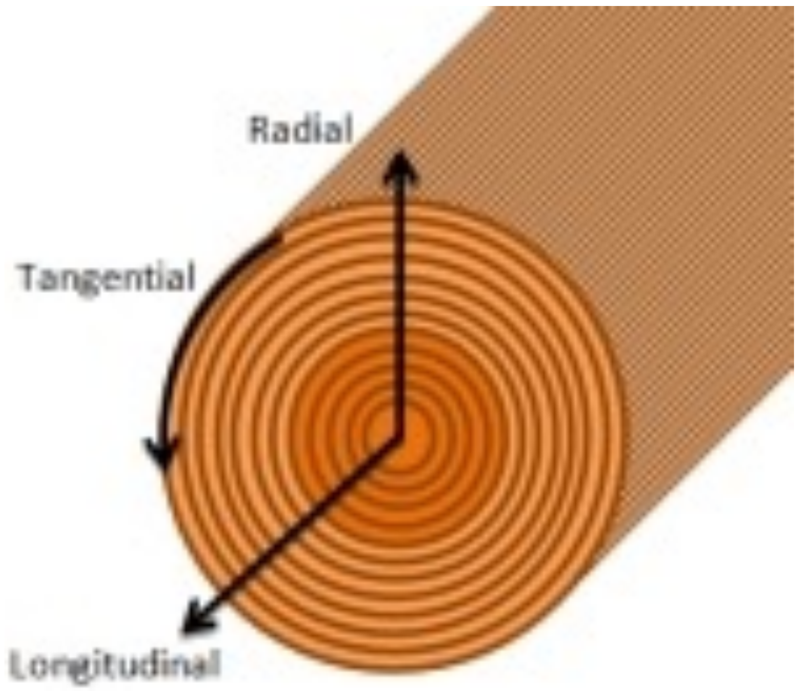
$$[\mathsf{C}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \equiv \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}$$

$$\sigma_i = C_{ij} \epsilon_j \ .$$

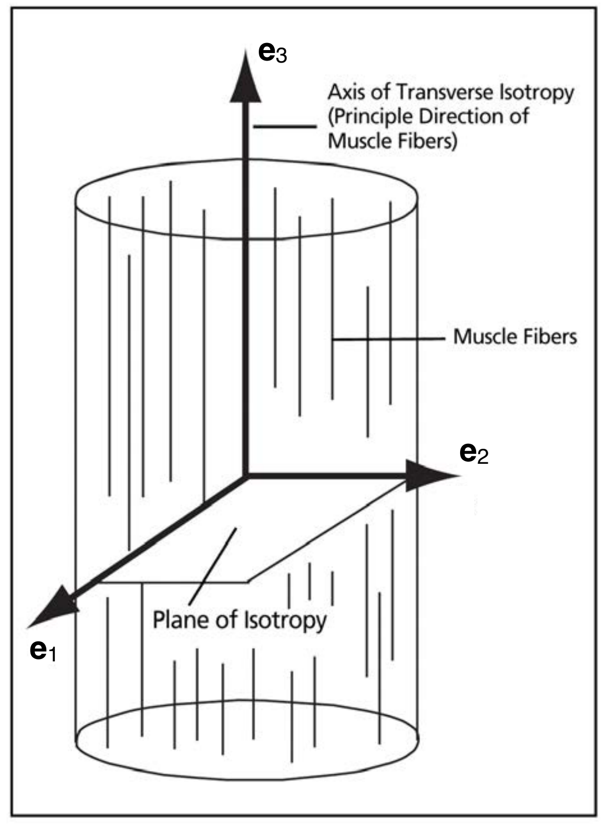
Hooke law in 3D (material symmetry)

Orthotropic
(3 plans)

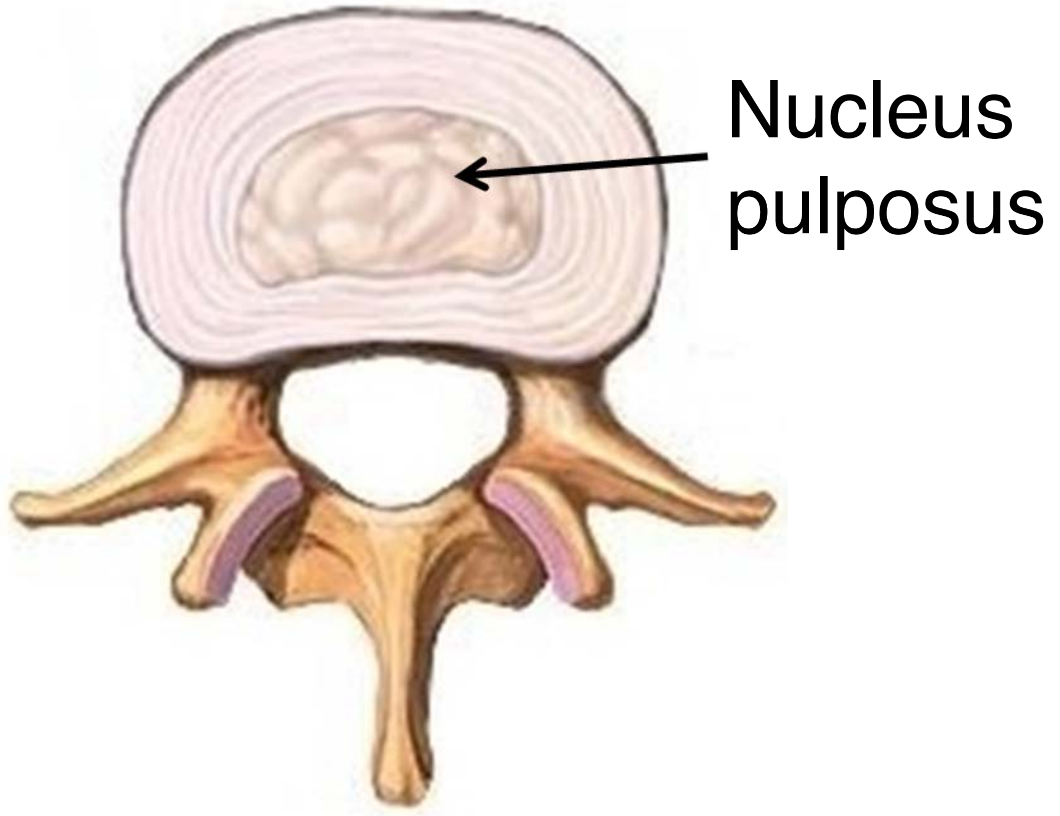


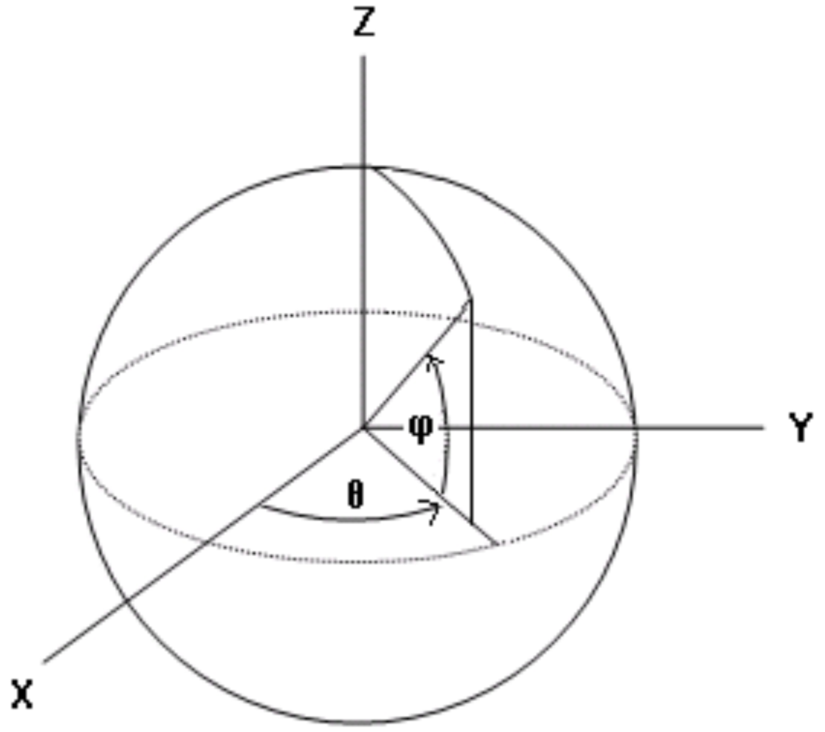


Transversely isotropic
(1 direction perpendicular to a plan)



Isotropic
(no symmetry)





Hooke law in 3D (material symmetry)

Isotropic material -> isotropic stiffness tensor (**2** parameters):

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

δ: Kronecker symbol

λ and μ are 2 scalars called <u>Lamé constants</u>

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \epsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

Tensorial formulation for linear elastic isotropic material

$$\boldsymbol{\sigma} = \lambda(\mathrm{tr}\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}$$

$$\mathrm{tr}\boldsymbol{\varepsilon} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$\mathbf{I}$: tensor identity

→ link with "usual" E (Young's modulus)

$\sigma_{33} = E\varepsilon_{33}$?

Hooke law in 3D (material symmetry -> **isotropy**)

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \epsilon_5 \\ \varepsilon_6 \end{bmatrix} = 1/E \begin{bmatrix} 1 & -\upsilon & -\upsilon & 0 & 0 & 0 \\ -\upsilon & 1 & -\upsilon & 0 & 0 & 0 \\ -\upsilon & -\upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\upsilon) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\upsilon) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\upsilon) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

Tensorial formulation -> $\boldsymbol{\varepsilon} = {}^{1}/_{E}\,(\boldsymbol{\sigma} - \upsilon[tr(\sigma)\boldsymbol{I} - \boldsymbol{\sigma}])$

Voigt notation (3) -> $\varepsilon_3 = {}^{1}/_{E}\,[\sigma_3 - \upsilon(\sigma_1 + \sigma_2)]$

Index notation (33) -> $\varepsilon_{33} = {}^{1}/_{E}\,[\sigma_{33} - \upsilon(\sigma_{11} + \sigma_{22})]$

Relation between the different isotropic elastic linear parameters

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

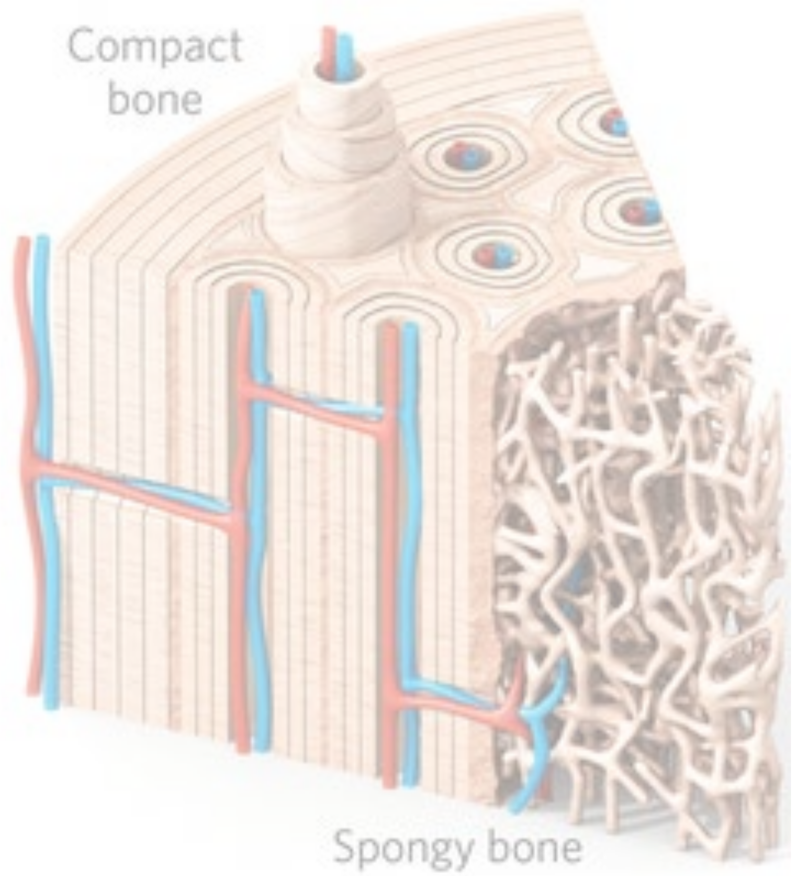
Relation between the different isotropic elastic linear parameters

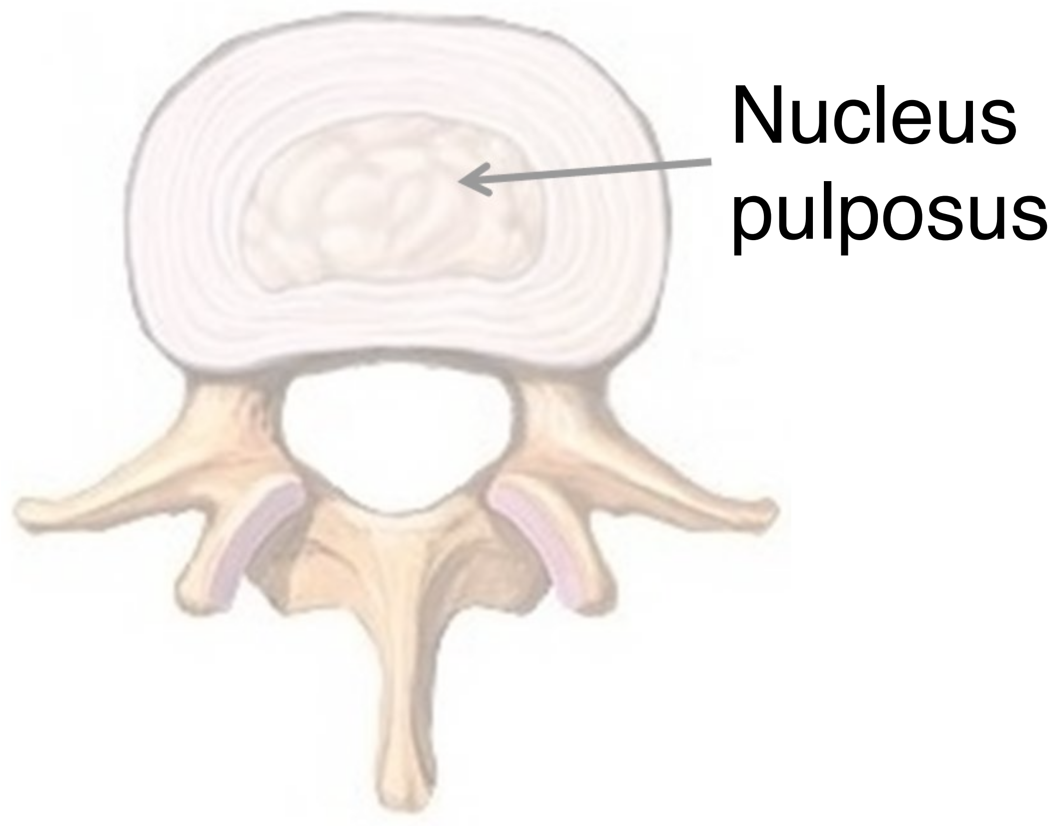
	λ	μ	E	ν	k
λ, μ	–	–	$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$	$\frac{3\lambda+2\mu}{3}$
λ, ν	–	$\frac{\lambda(1-2\nu)}{2\nu}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	–	$\frac{\lambda(1+\nu)}{3\nu}$
λ, k	–	$\frac{3(k-\lambda)}{2}$	$\frac{9k(k-\lambda)}{3k-\lambda}$	$\frac{\lambda}{3k-\lambda}$	–
μ, E	$\frac{(2\mu-E)\mu}{(E-3\mu)}$	–	–	$\frac{(E-2\mu)}{2\mu}$	$\frac{\mu E}{3(3\mu-E)}$
μ, ν	$\frac{2\mu\nu}{1-2\nu}$	–	$2\mu(1+\nu)$	–	$\frac{2\mu(1+\nu)}{3(1-2\nu)}$
μ, k	$\frac{3k-2\mu}{3}$	–	$\frac{9k\mu}{3k+\mu}$	$\frac{3k-2\mu}{6k+2\mu}$	–
E, ν	$\frac{\nu E}{(1+\nu)(1-2\nu)}$	$\frac{E}{2(1+\nu)}$	–	–	$\frac{E}{3(1-2\nu)}$
E, k	$\frac{3k(3k-E)}{9k-E}$	$\frac{3kE}{9k-E}$	–	$\frac{(3k-E)}{6k}$	–
ν, k	$\frac{3k\nu}{1+\nu}$	$\frac{3k(1-2\nu)}{2(1+\nu)}$	$3k(1-2\nu)$	–	–

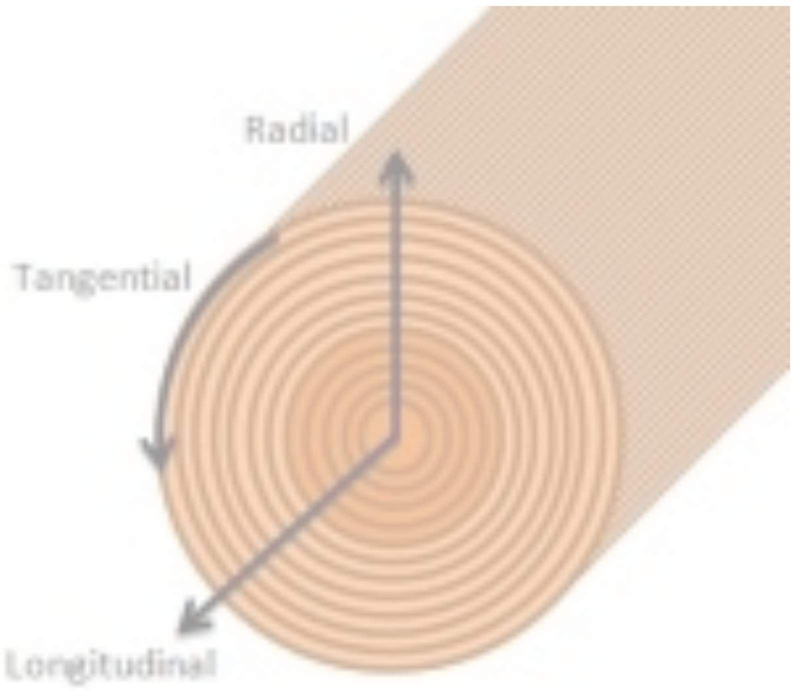
Tissue mechanics, Cowin, 2007

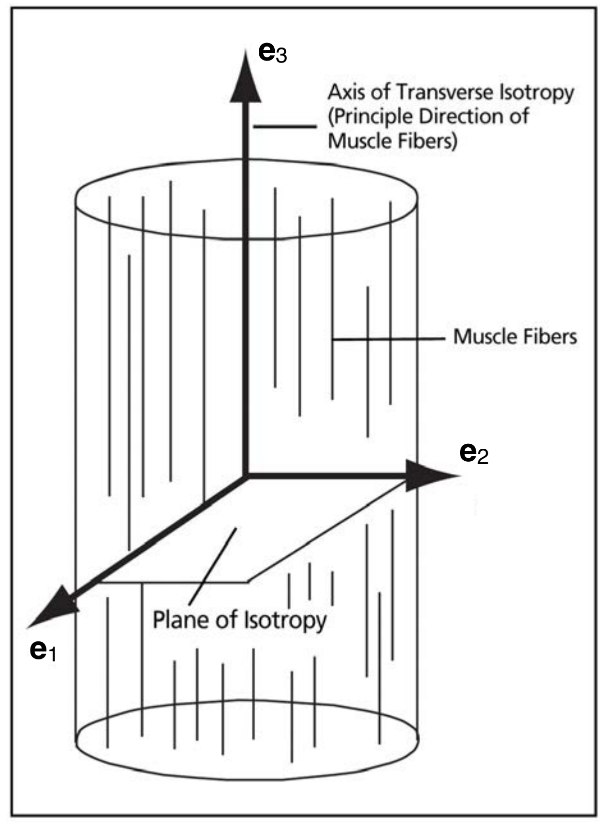
Hooke law in 3D (material symmetry)

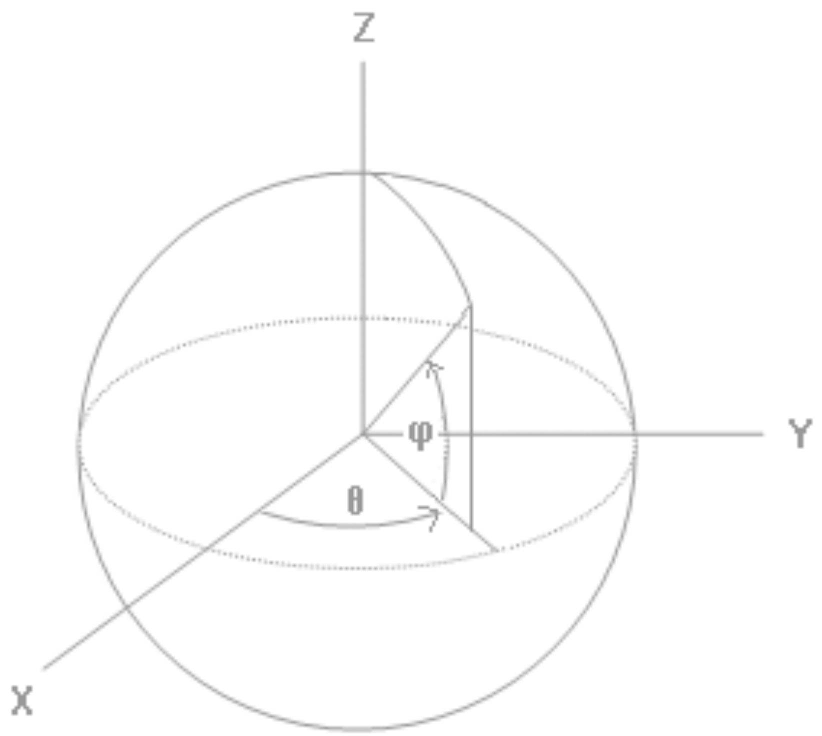
Orthotropy	Transverse isotropy	Isotropy
(3 plans)	(1 direction perpendicular to a plan)	(no symmetry)











Hooke law in 3D (material symmetries -> **transversely isotropic**)

Material symmetry (anisotropy e.g transverse isotropy -> **5** parameters)

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \epsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{E_p} & \frac{-\nu_p}{E_p} & \frac{-\nu_t}{E_p} & 0 & 0 & 0 \\ \frac{-\nu_p}{E_p} & \frac{1}{E_p} & \frac{-\nu_t}{E_p} & 0 & 0 & 0 \\ \frac{-\nu_t}{E_p} & \frac{-\nu_t}{E_p} & \frac{1}{E_t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{tp}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{tp}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\frac{1+\nu_p}{E_p} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

E_p, v_p: Young modulus and Poisson ratio in the plane of isotropy
E_t, v_t: Young modulus and Poisson ratio in the transverse direction (axis of symmetry)
G_{tp}: shear modulus in the plane of isotropy

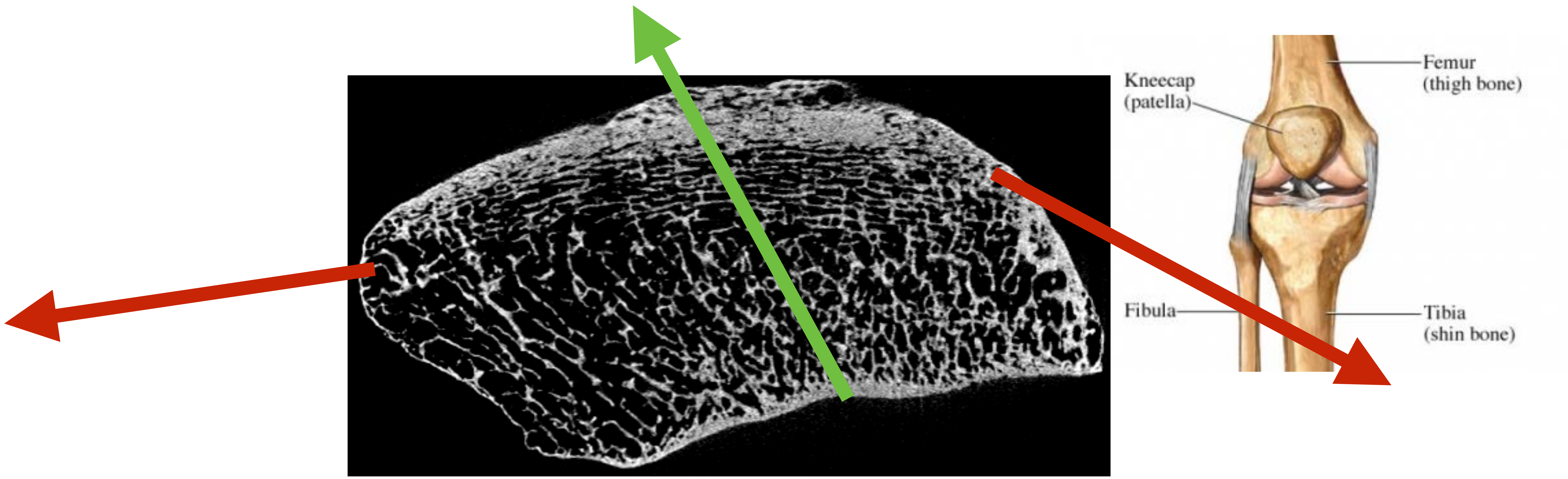
Material symmetries of different tissues

i) Isotropic -> cartilage ???

ii) Transverse isotropic -> ligament, tendon, bone?

iii) Orthotropic -> bone?

Material symmetries of different tissues



sagittal view

tendon force

femora-patella contact force

One important aspect of biomechanics is then to characterize tissues through constitutive laws

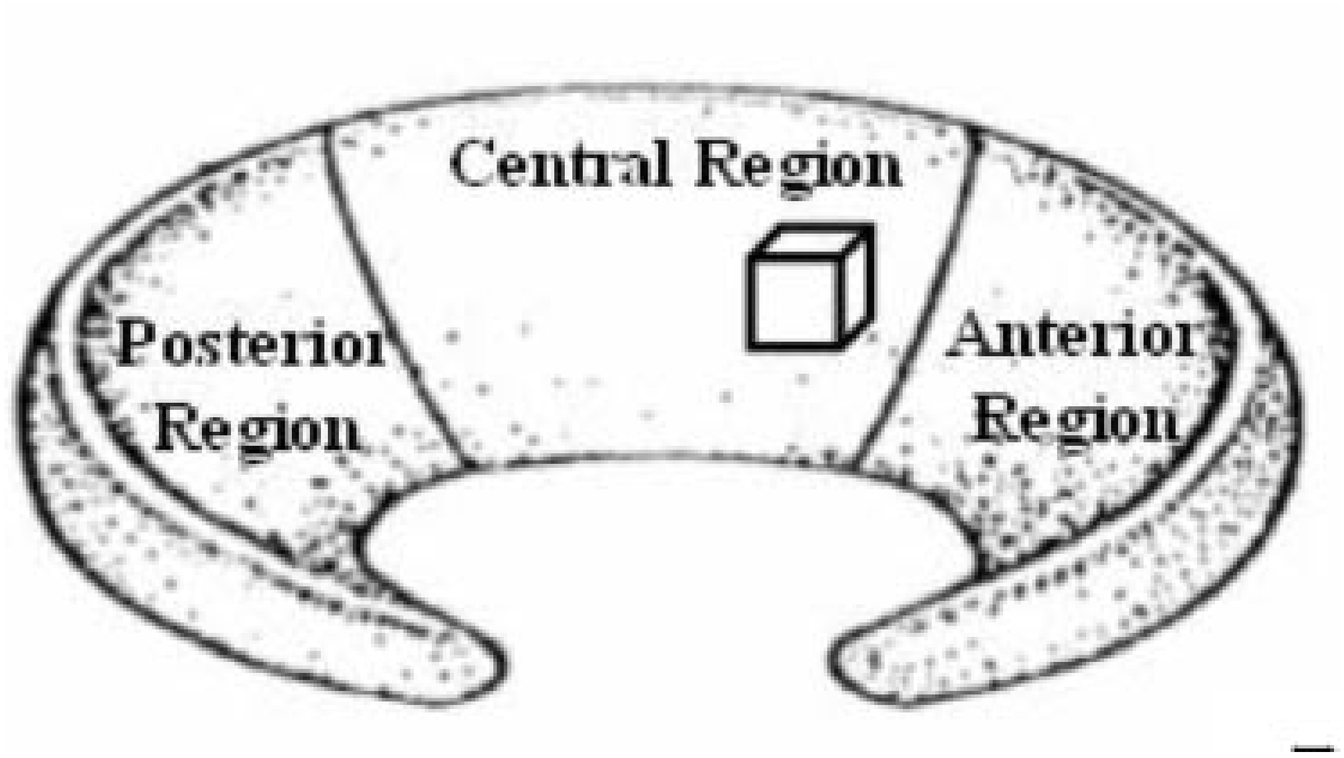
$$\boxed{\rho \frac{d\boldsymbol{v}}{dt} = div\, \boldsymbol{\sigma} + \rho \boldsymbol{b}}$$

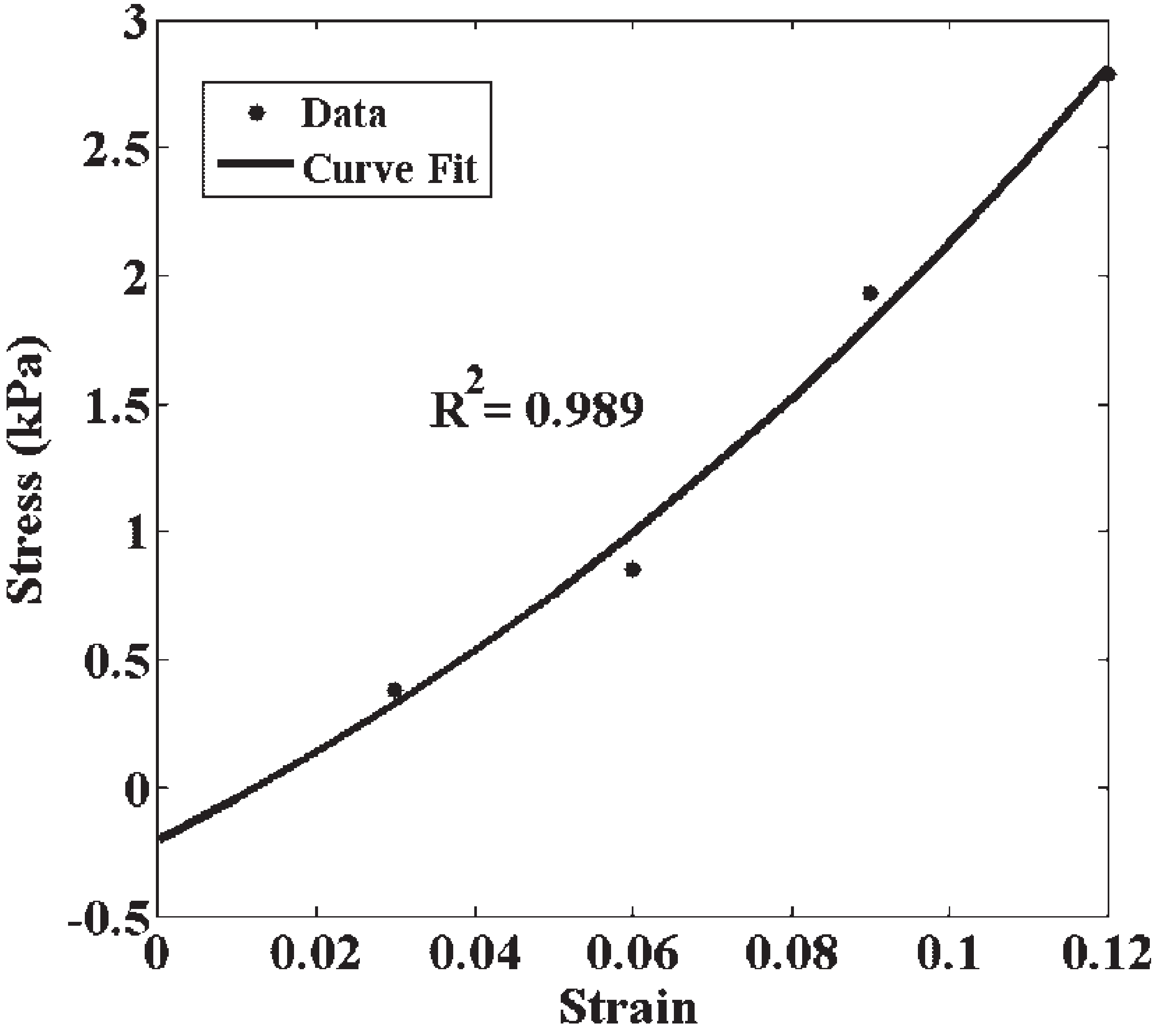
$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}, \varepsilon_p, \ldots)$$

Elasticity -> σ = σ(ε) → Linear

Elasticity -> σ = σ(ε) → $\boxed{\text{Non-linear}}$

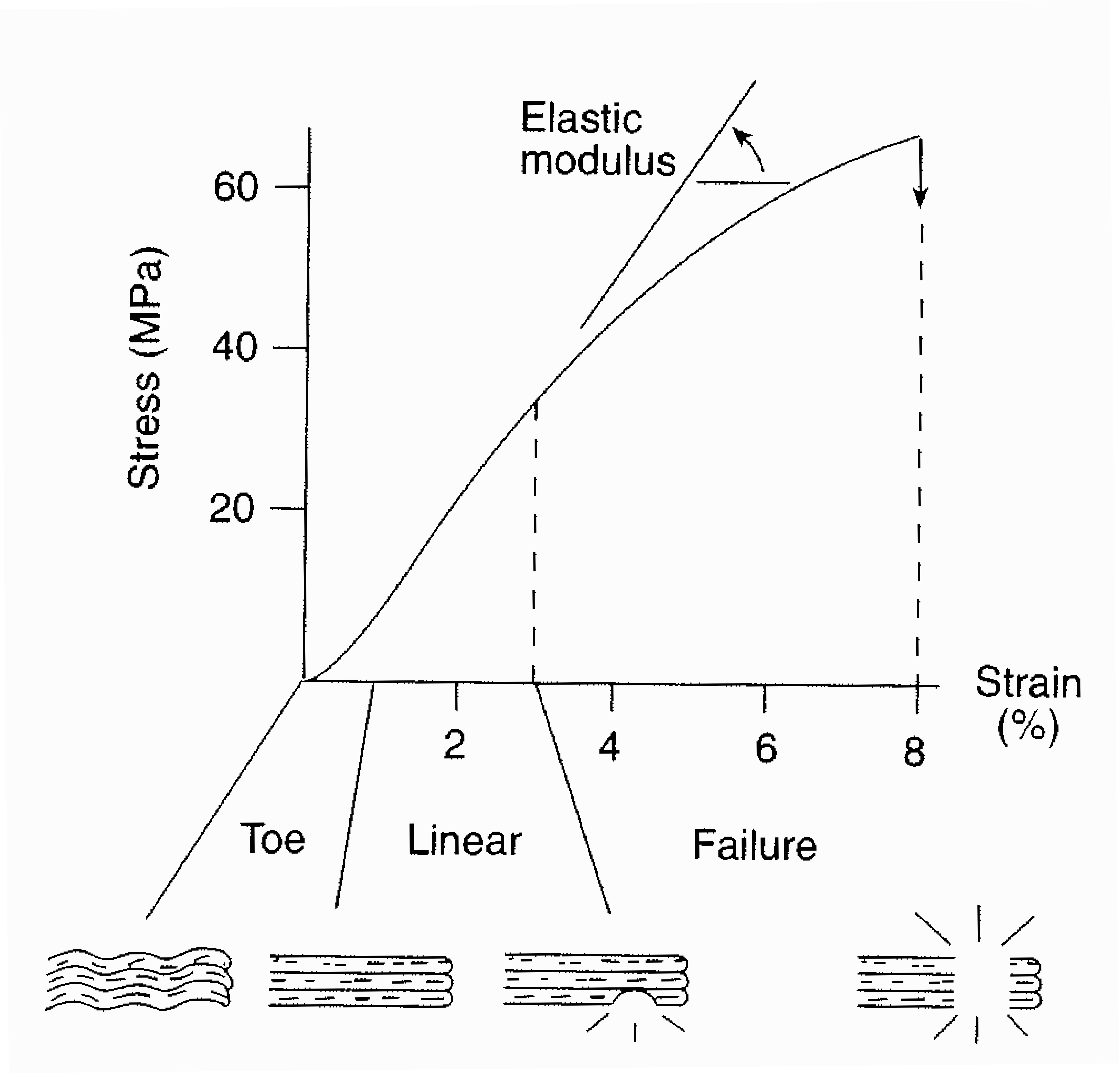
The compressive behaviour of the meniscus samples depend on their location and deformation





source: Helena and Hull, JOR, 2008

As well, the ligaments which work under traction, show a non-linear tensile behaviour

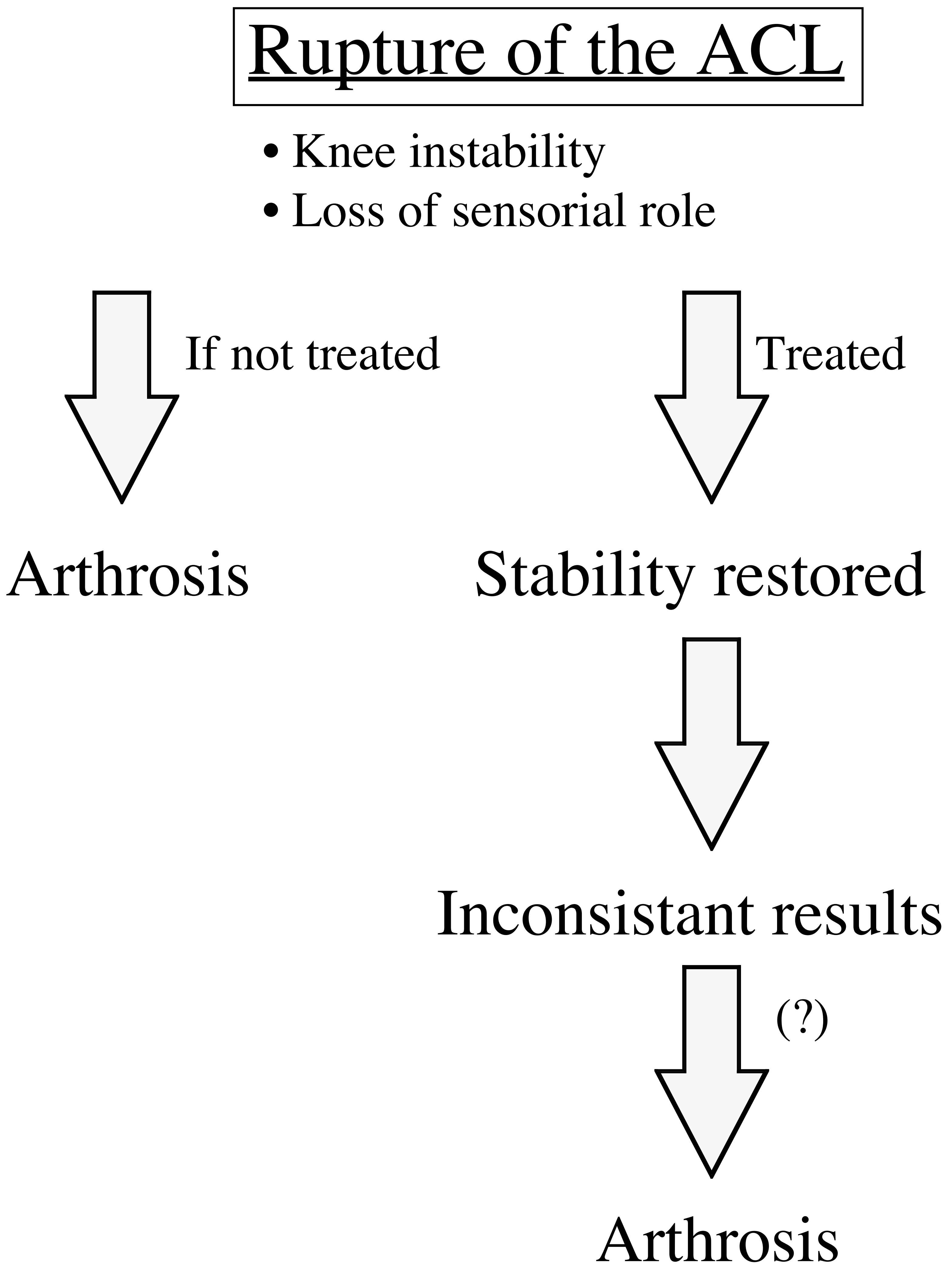


source: *Biomechanics of the musculoskeletal injury, W. Whiting and R. Zernicke, 1998*

Soft tissues biomechanics represent a challenge as these tissues have usually a non-linear mechanical behaviour

As a background, ACL rupture is frequent in the young and active population

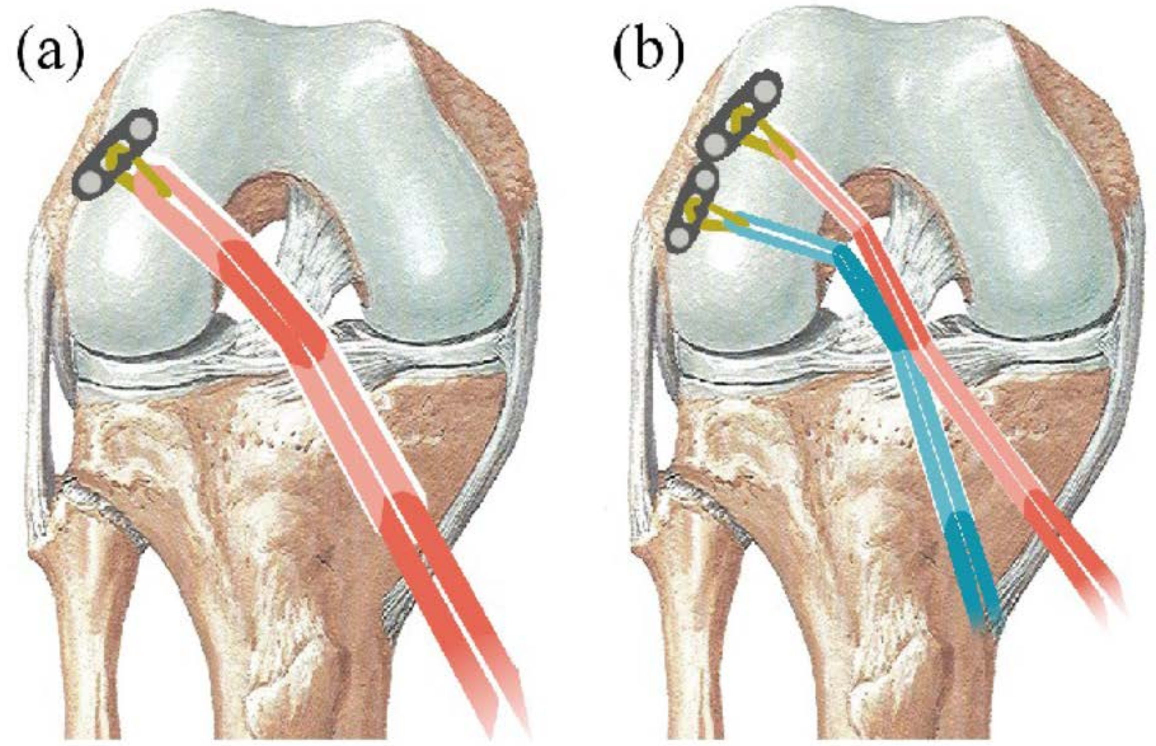
The choice of the treatment is not clearly defined



There are several surgical approaches for the treatment

<u>Techniques</u>

- Over-the-top, Macintosh
- Simple (a), double (b)
- Standard, arthroscopy



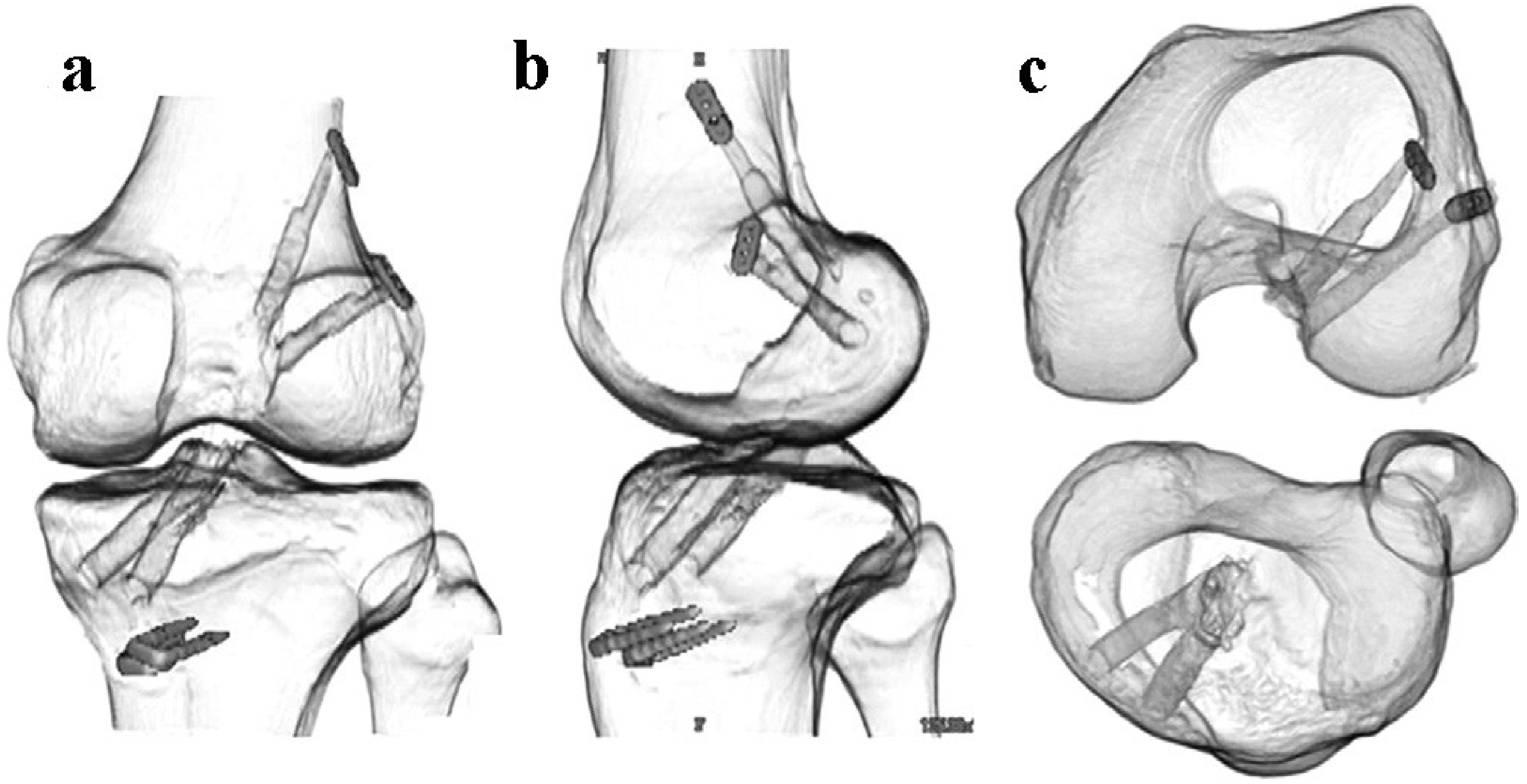
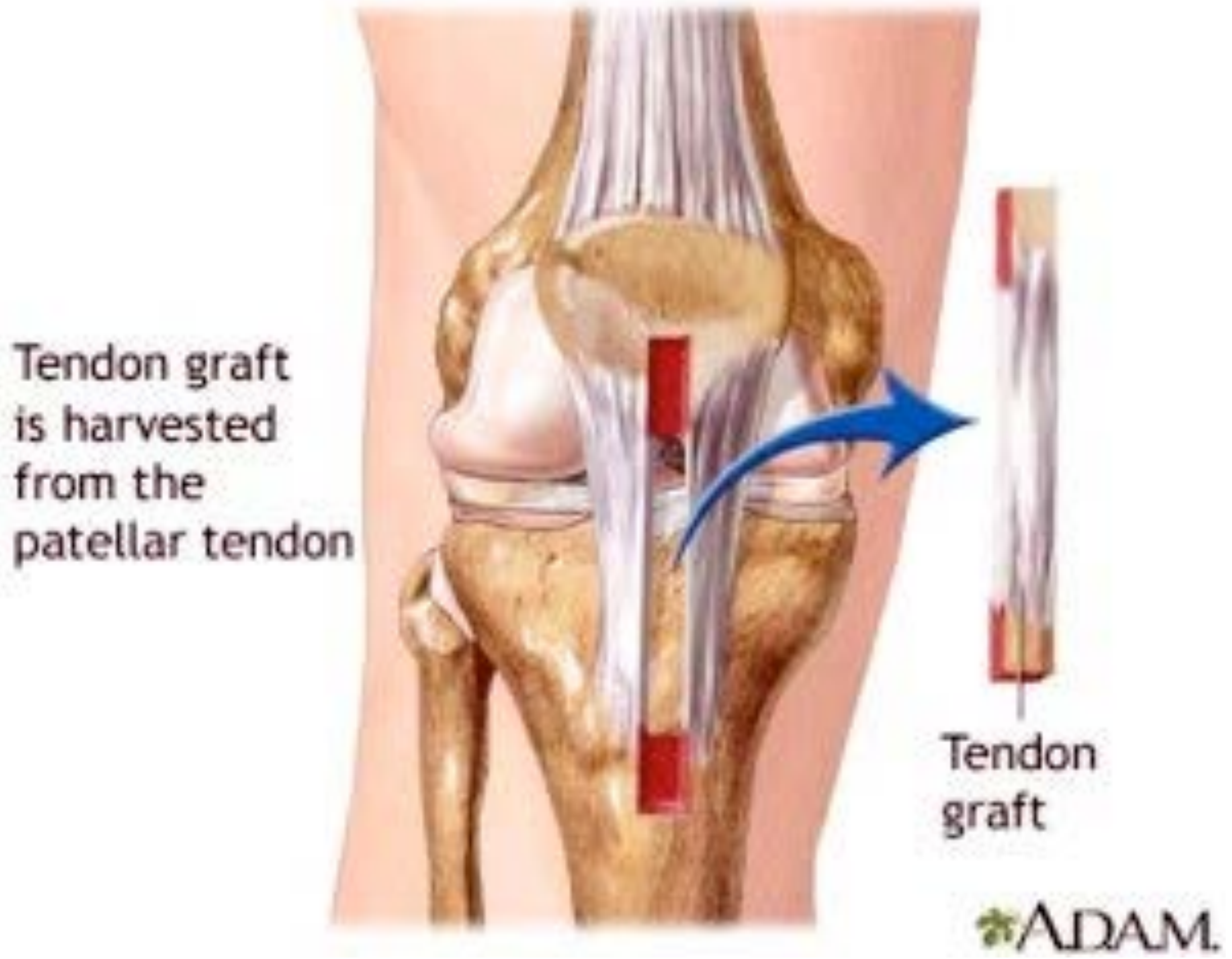


Illustration I 12: Visualisation des tunnels et des fixations d'une reconstruction à double faisceau obtenues à partir de résultats de tomographie 3D. (a) vue postérieure (b) vue sagittale (c) vue dans le plan transversal.

The type of grafts and the rehabilitation programs are also diverse



Type of grafts

- Auto, allo or artificial grafts
- Patellar tendon, semi-tendinous

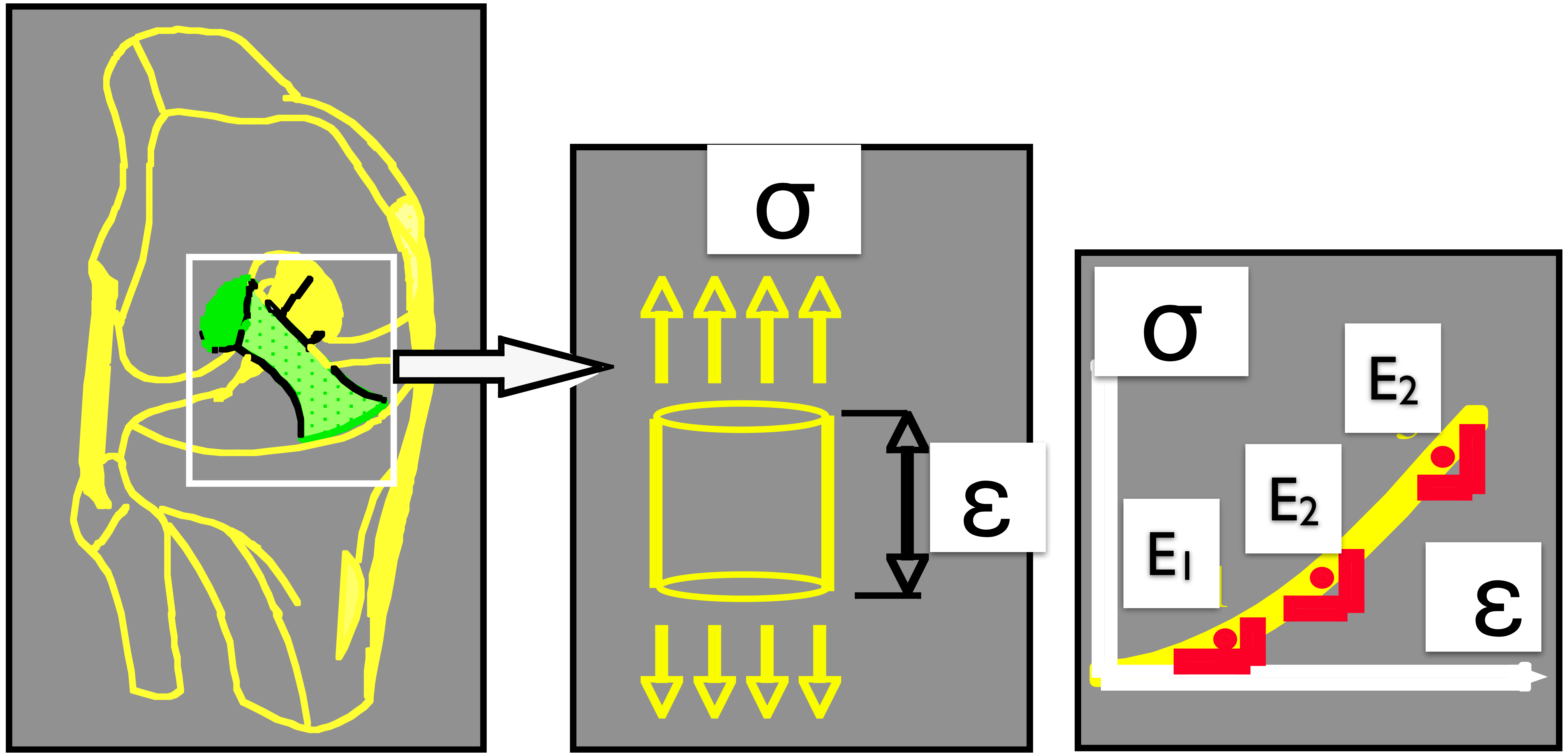
Rehabilitation

- Rapid mobility, rest
- Partial, complete mobility
- Use of brace, tape

A biomechanical description of the ligament is then useful for divers reasons

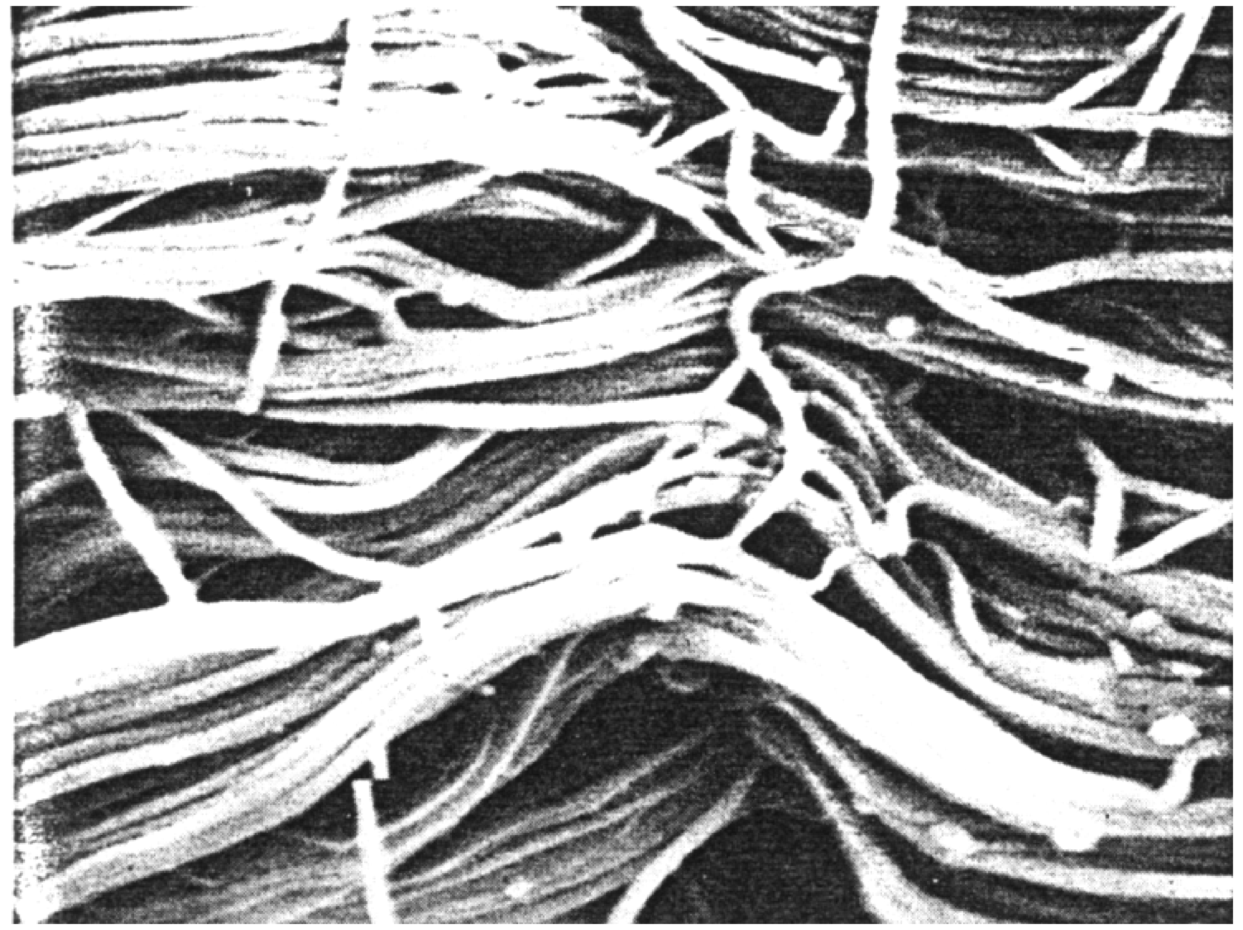
- Mechanical role of the ligament
- Kinematics of the knee
- Global model of the knee
- Improvement of surgical technique
- Input for a biological description

We want to perform mechanical tests on a ligament to obtain a “stress-strain” curve



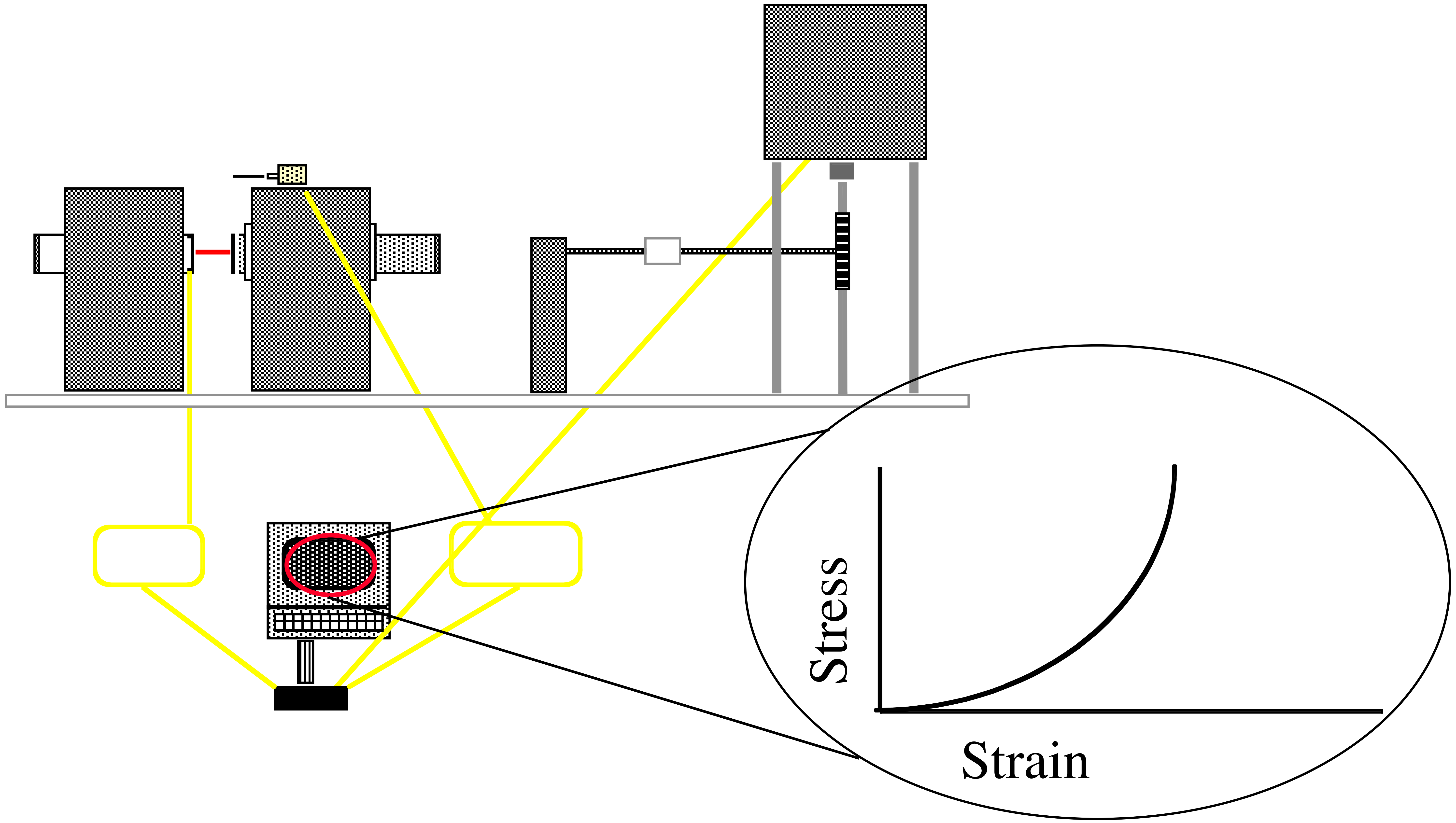
We first have to evaluate the parameters which may influence the stress-strain curves

- age
- sex
- temperature
- hydration
- conservation mode
- orientation
- ...

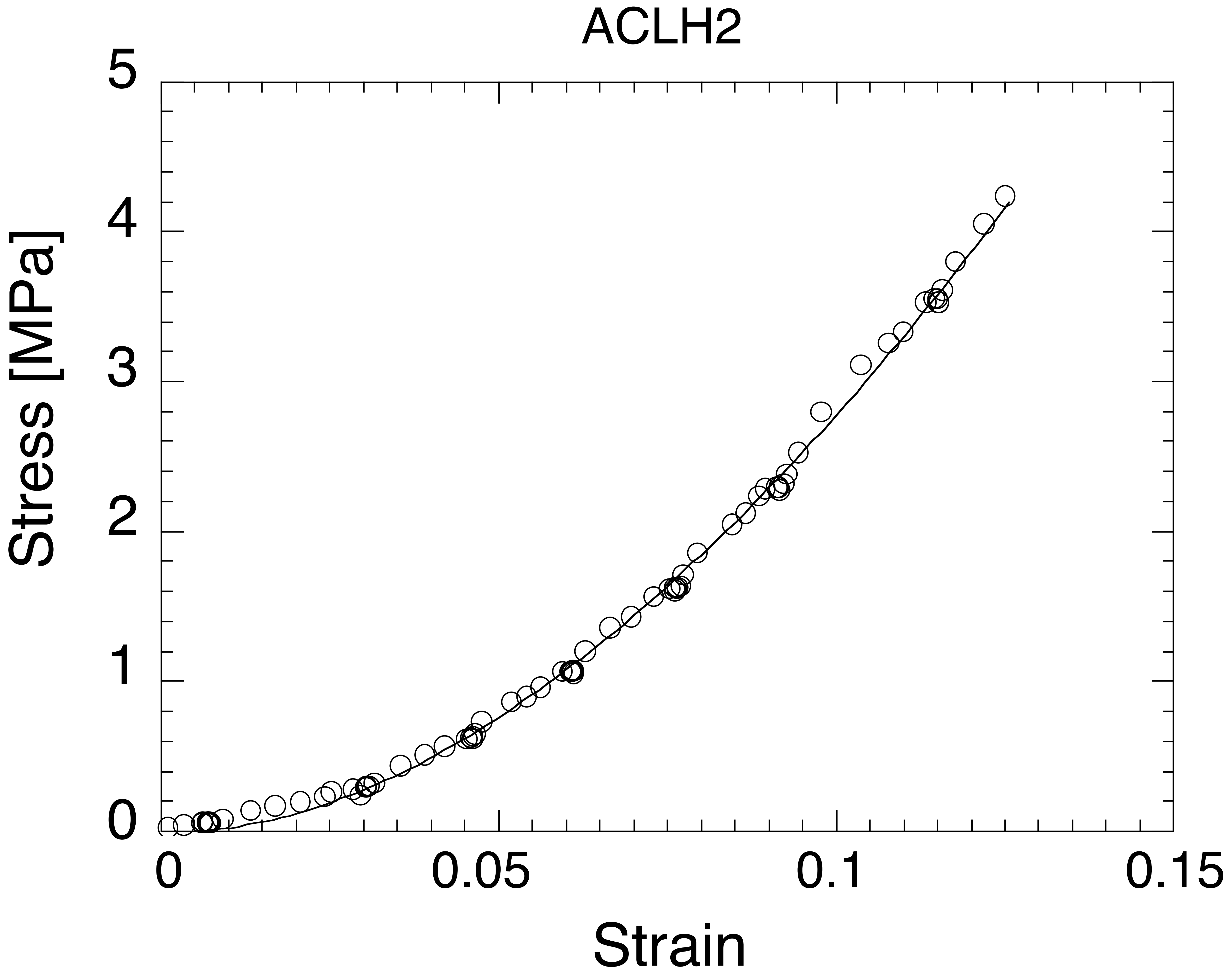


EM image highlighting
the importance of the specimen
orientation before performing
a biomechanical test

Stress-strain curves are experimentally obtained



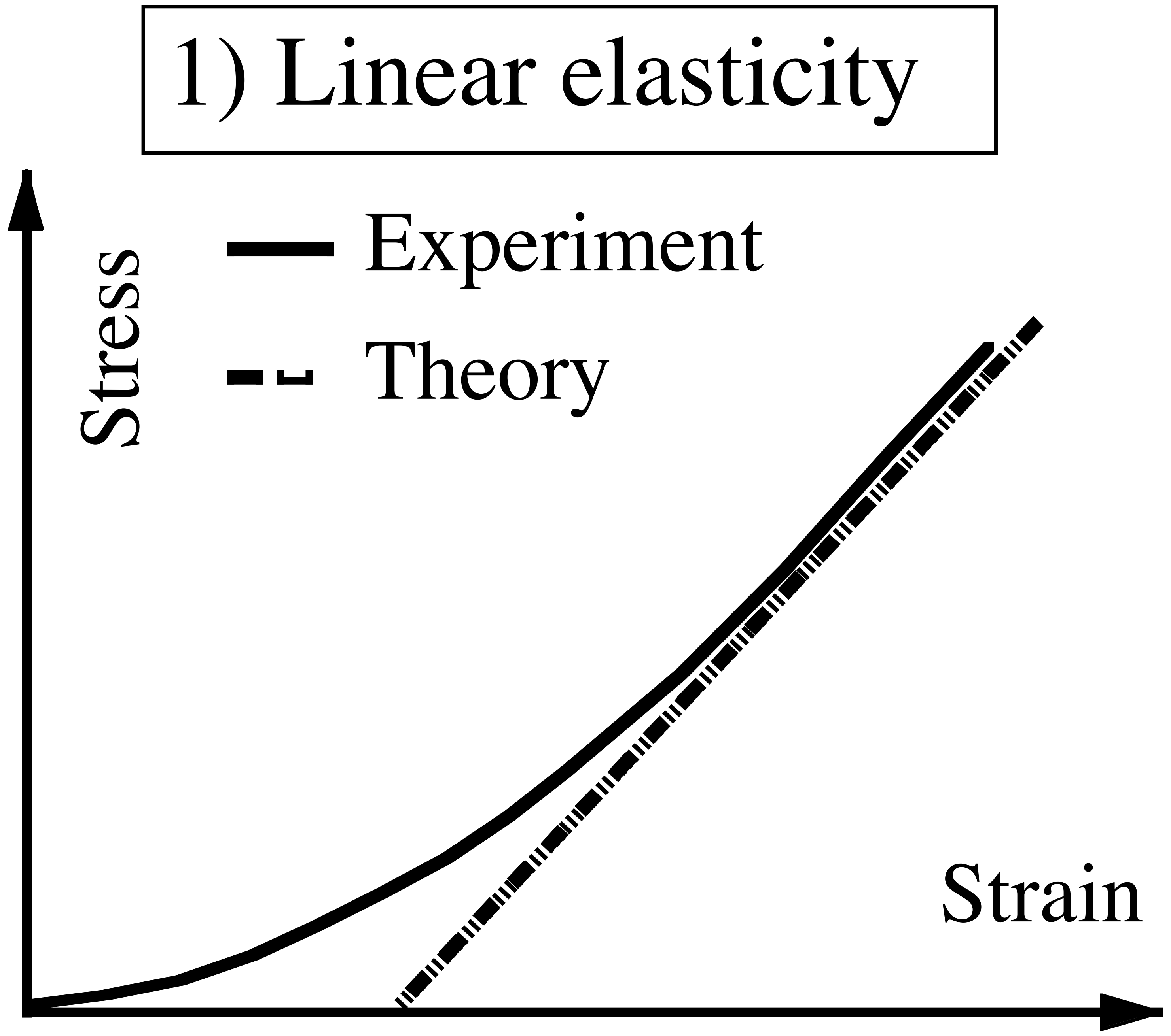
Experimental stress-strain curve of a human ACL specimen



Identification theory-experiment

1) Linear elasticity
2) Non-linear elasticity

=> $\sigma = \sigma(\varepsilon)$

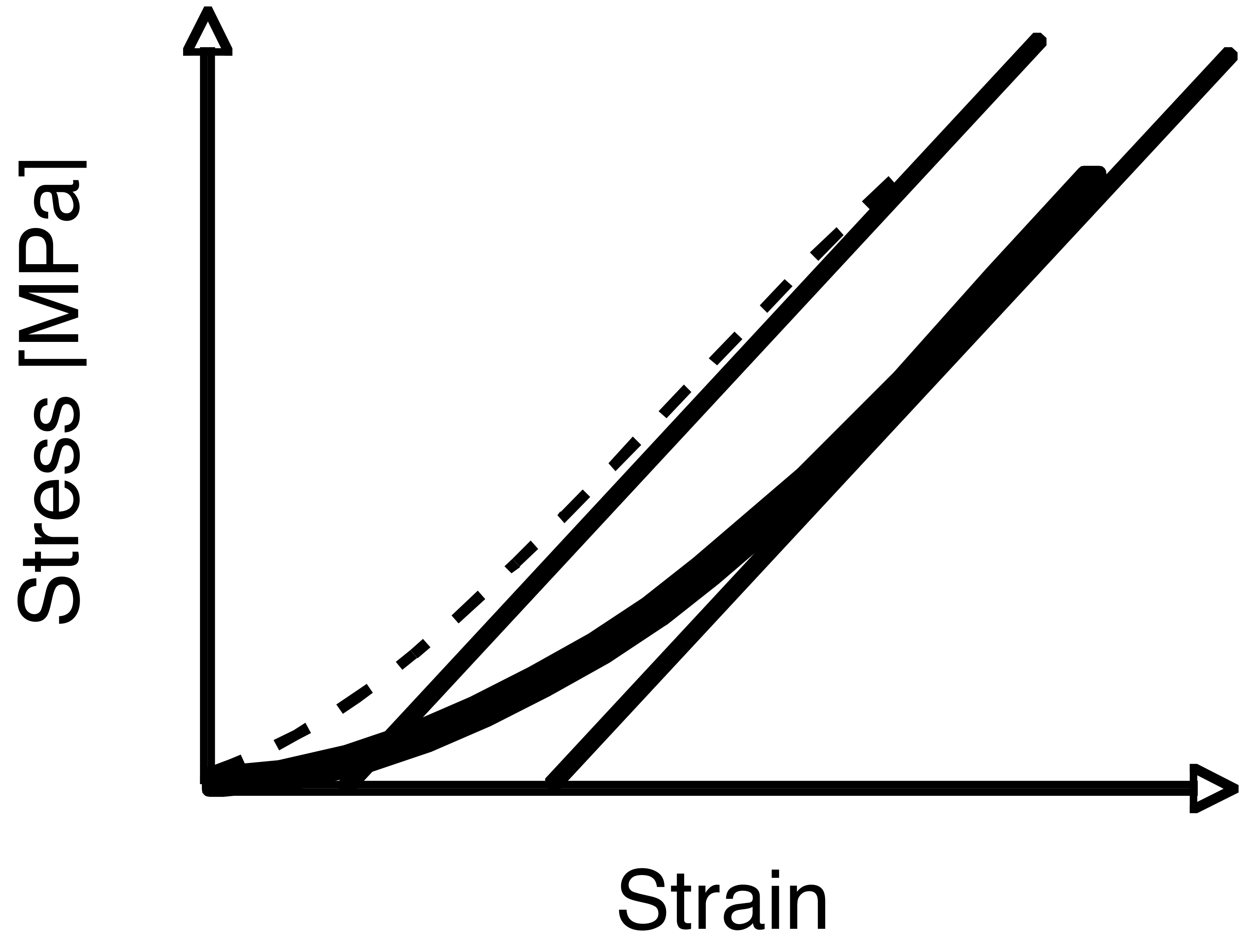


Elastic modulus: one figure (Young modulus)

Human PCL:

- Butler, 1986: 345 ± 107 MPa
- Race, 1994: 248 ± 119 MPa

The linear elastic description is restrictive



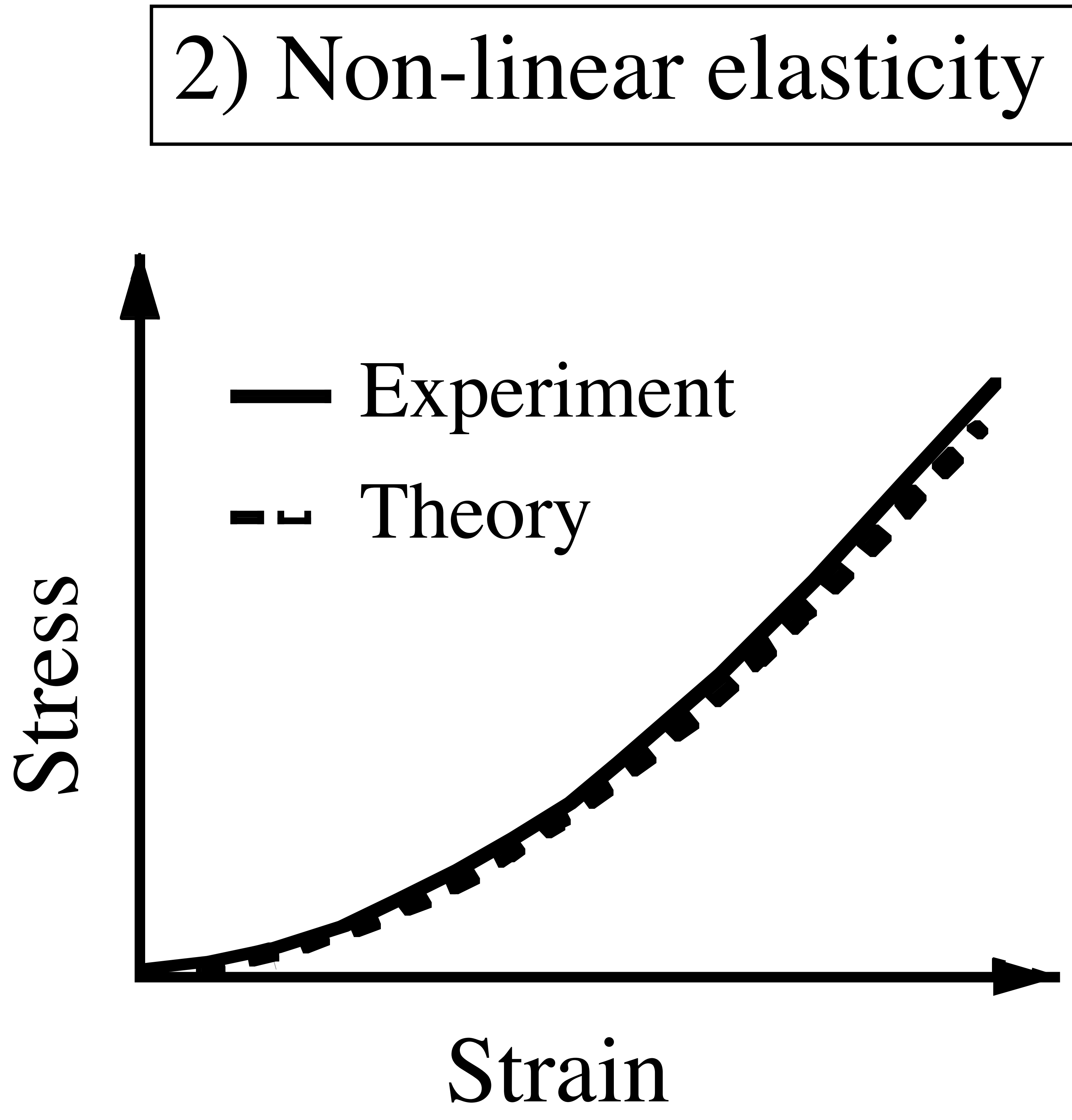
Patellar tendon
Young modulus

Dog	Human
360 MPa	337 MPa
(Burks, 1990)	(Flahiff, 1994)

curve 1 ≠ curve 2

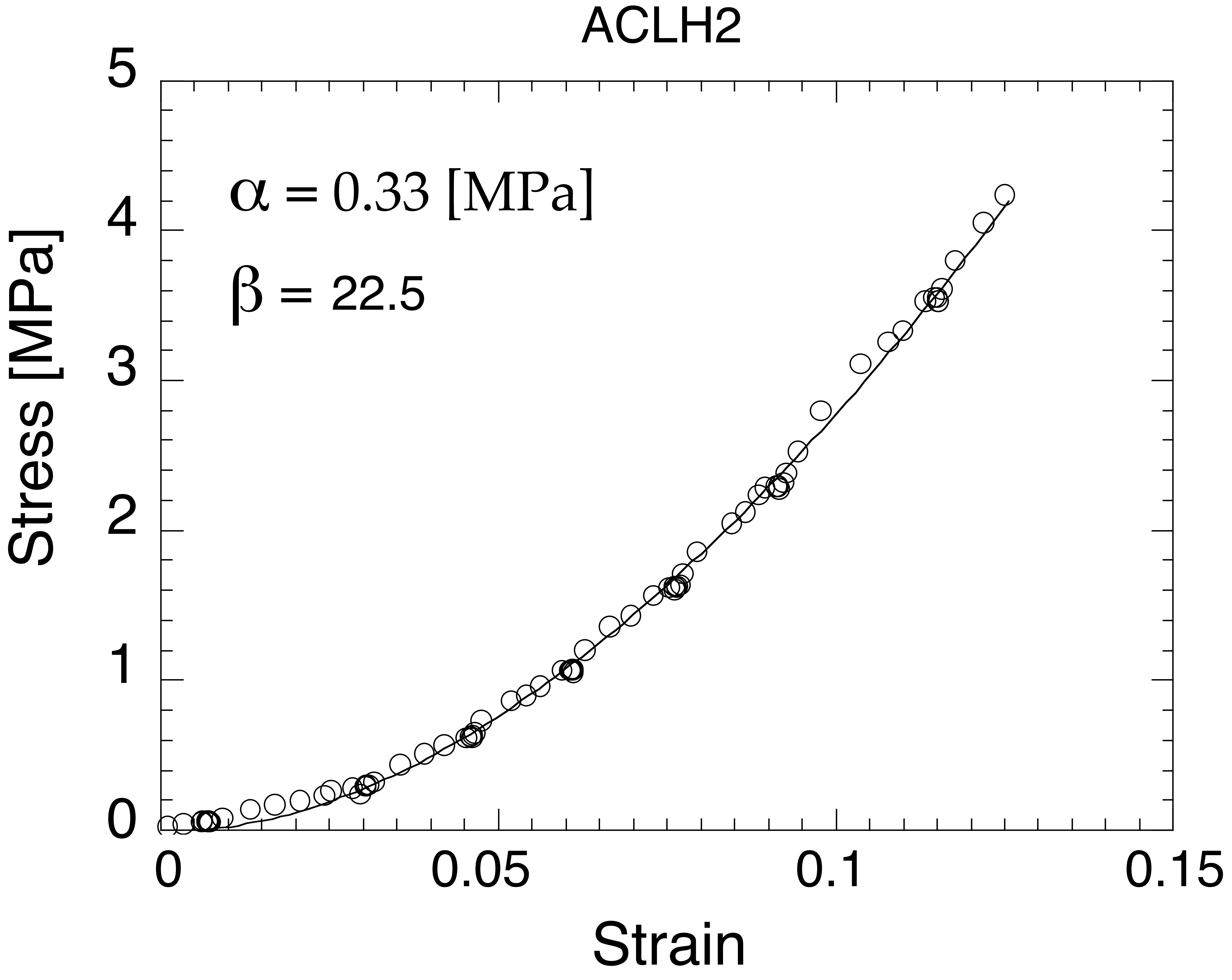
Young modulus 1 = Young modulus 2

The non-linear elastic description allows to describe the entire stress-strain curve



Elastic modulus:
mathematical function

The stress-strain curve is described through an exponential function



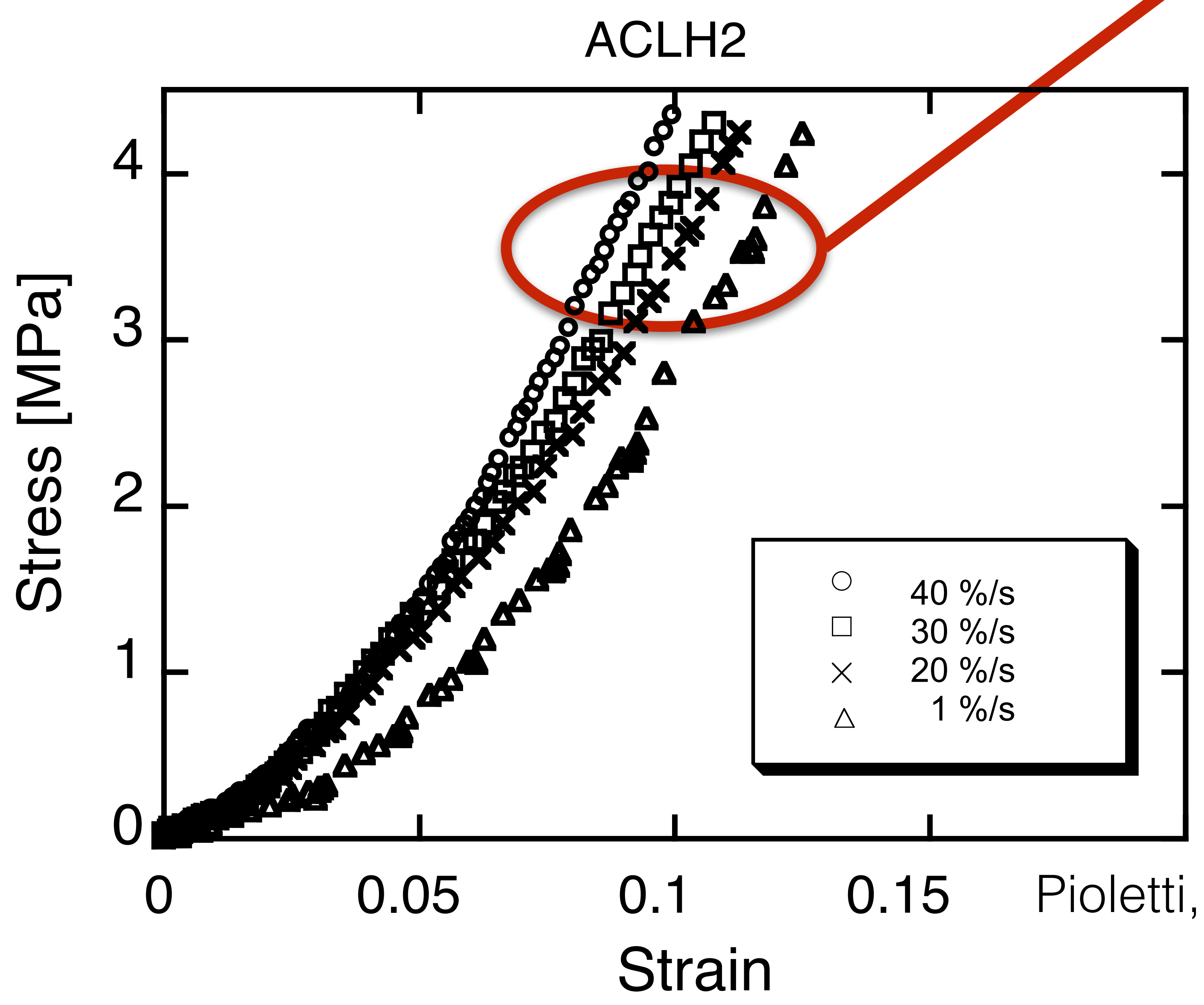
$$\sigma = \alpha(e^{\beta\varepsilon} - 1)$$

source: D. Pioletti, Europ J Mechanics, 2000

Biomechanical aspects are involved in most musculoskeletal conditions

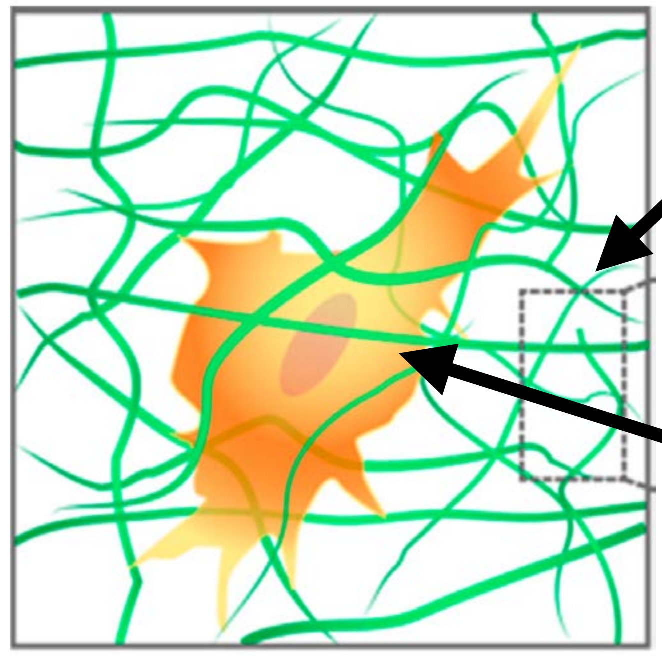
Mechanical

Viscoelasticity



Pioletti, 1997

Biological



Extra cellular matrix (ECM)

Cell

?

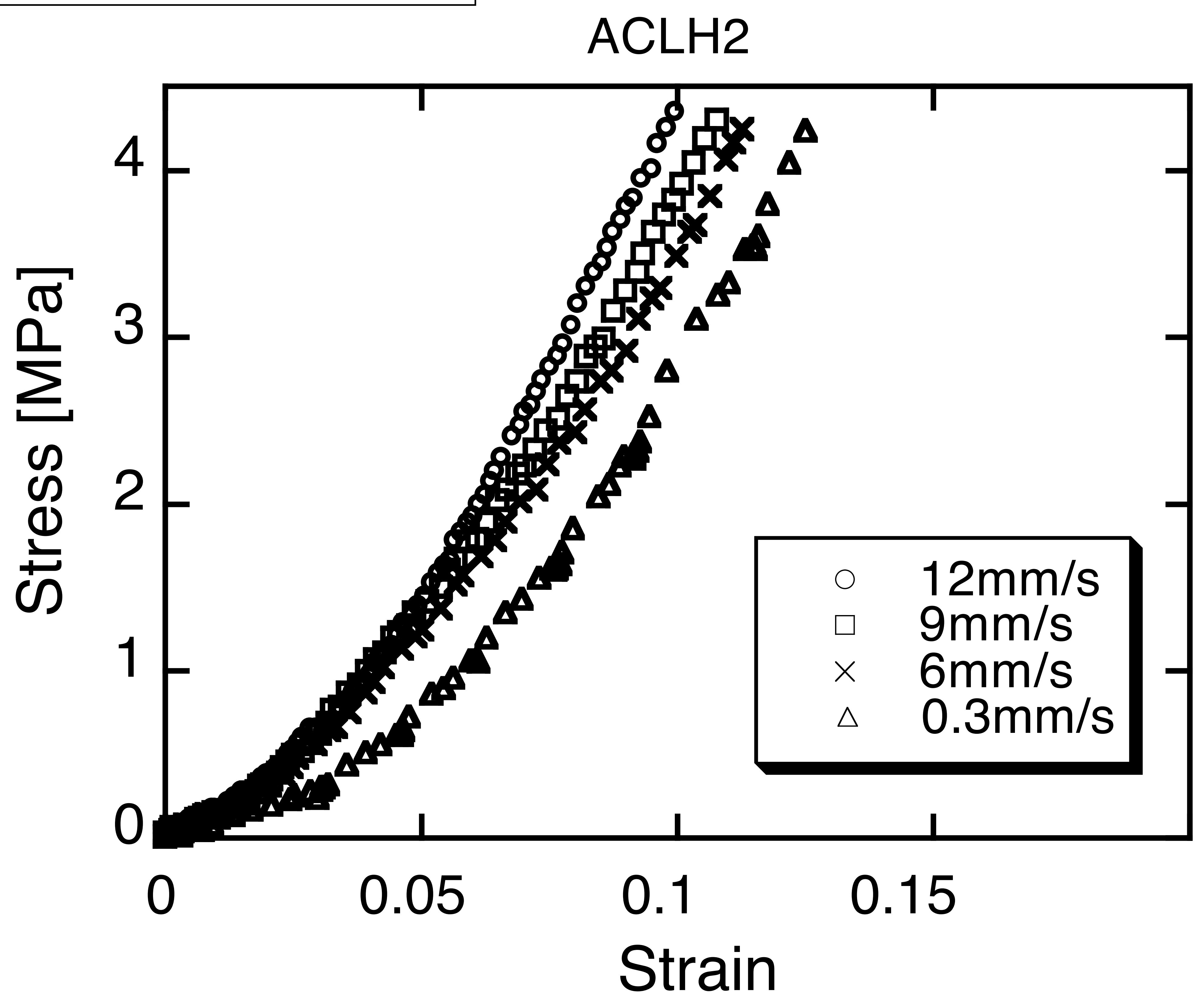
Traction tests performed at different strain rates highlight the viscoelastic behaviour of the ligament

1) Linear elasticity
2) Non-linear elasticity

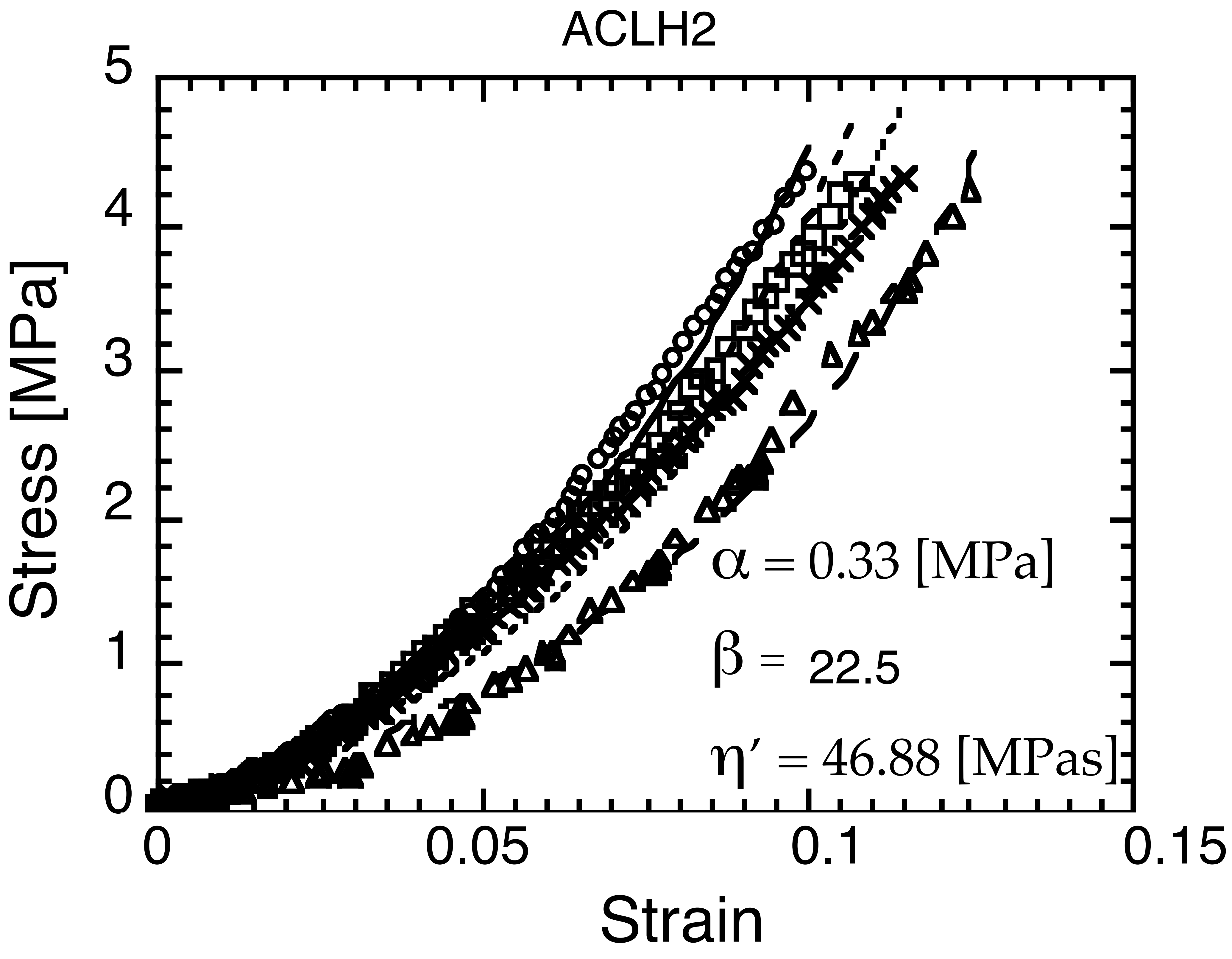
} Not enough to explain the observed ligament rupture

→ 3) Non-linear Viscoelasticity

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon})$$



The viscous part is determined on the curves obtained at different strain rates



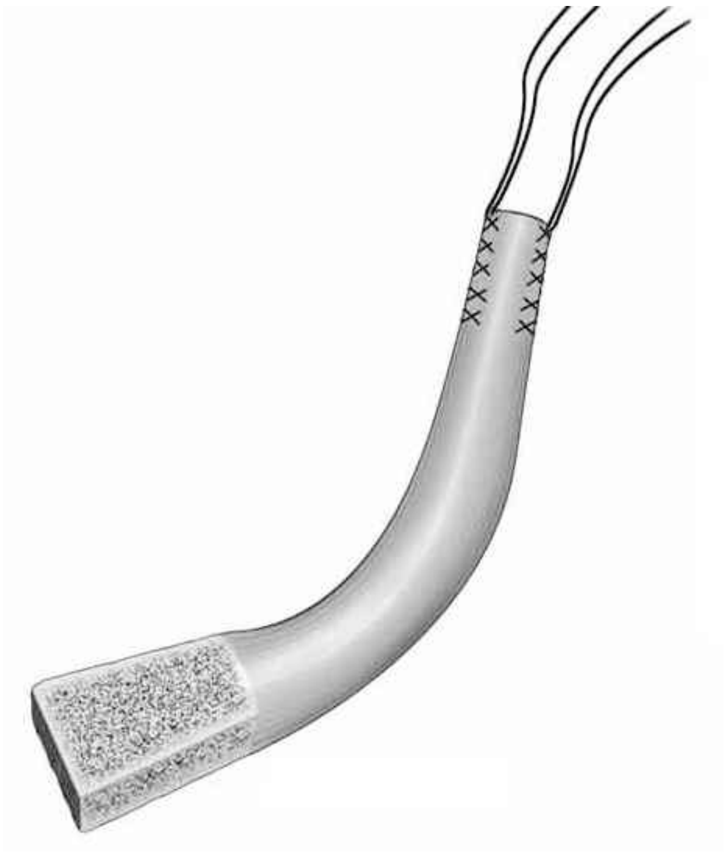
$$\sigma = \alpha(e^{\beta\varepsilon} - 1) + \eta'\dot{\varepsilon}$$

Soft tissues biomechanics represent a challenge as these tissues have usually a non-linear mechanical behaviour

As a background, ACL rupture is frequent in the young and active population

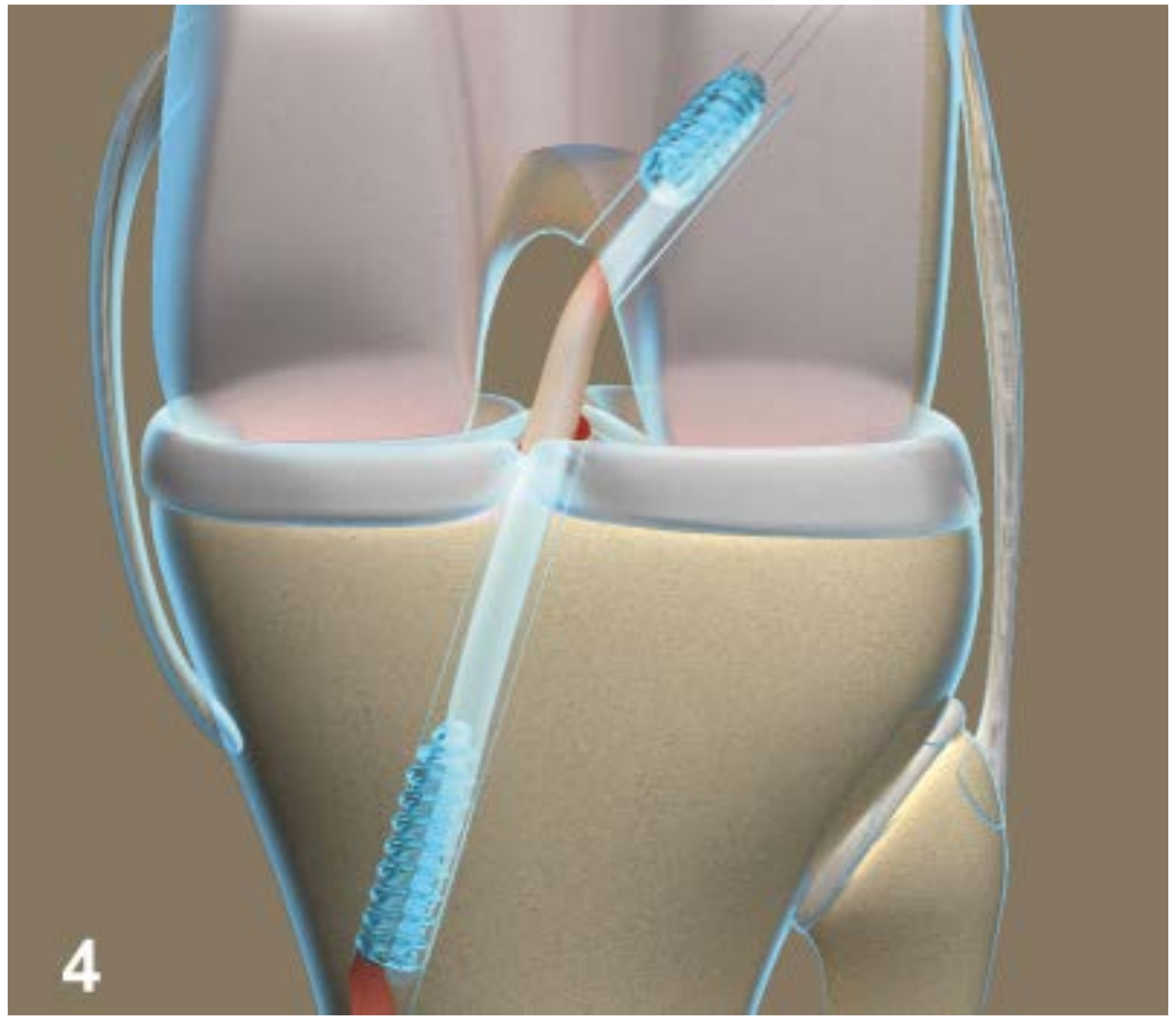
Once the rupture of the ligament is confirmed, it may be necessary to repair it

-> ligamentoplasty

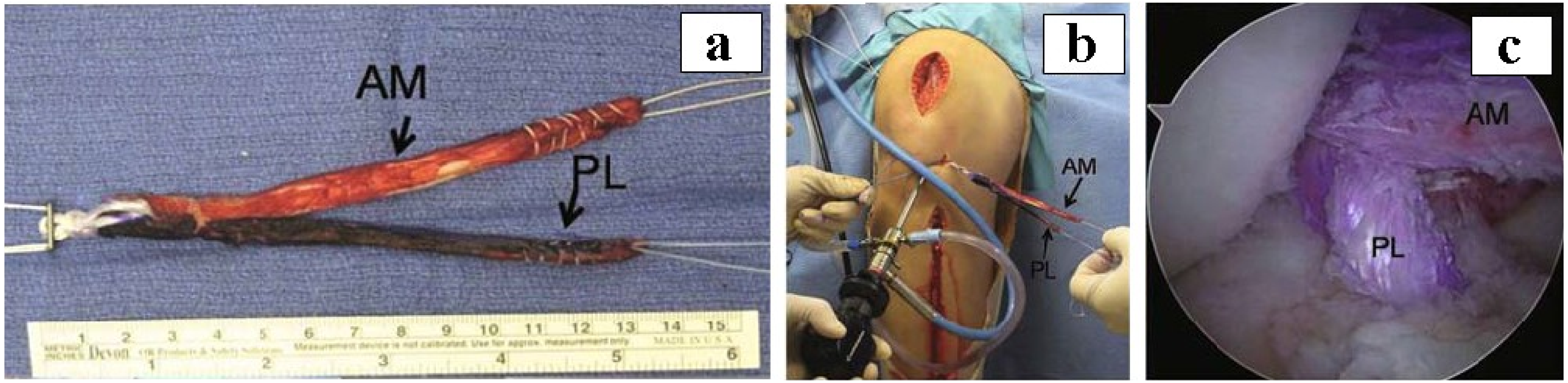


One-bundle graft

Double-bundle graft

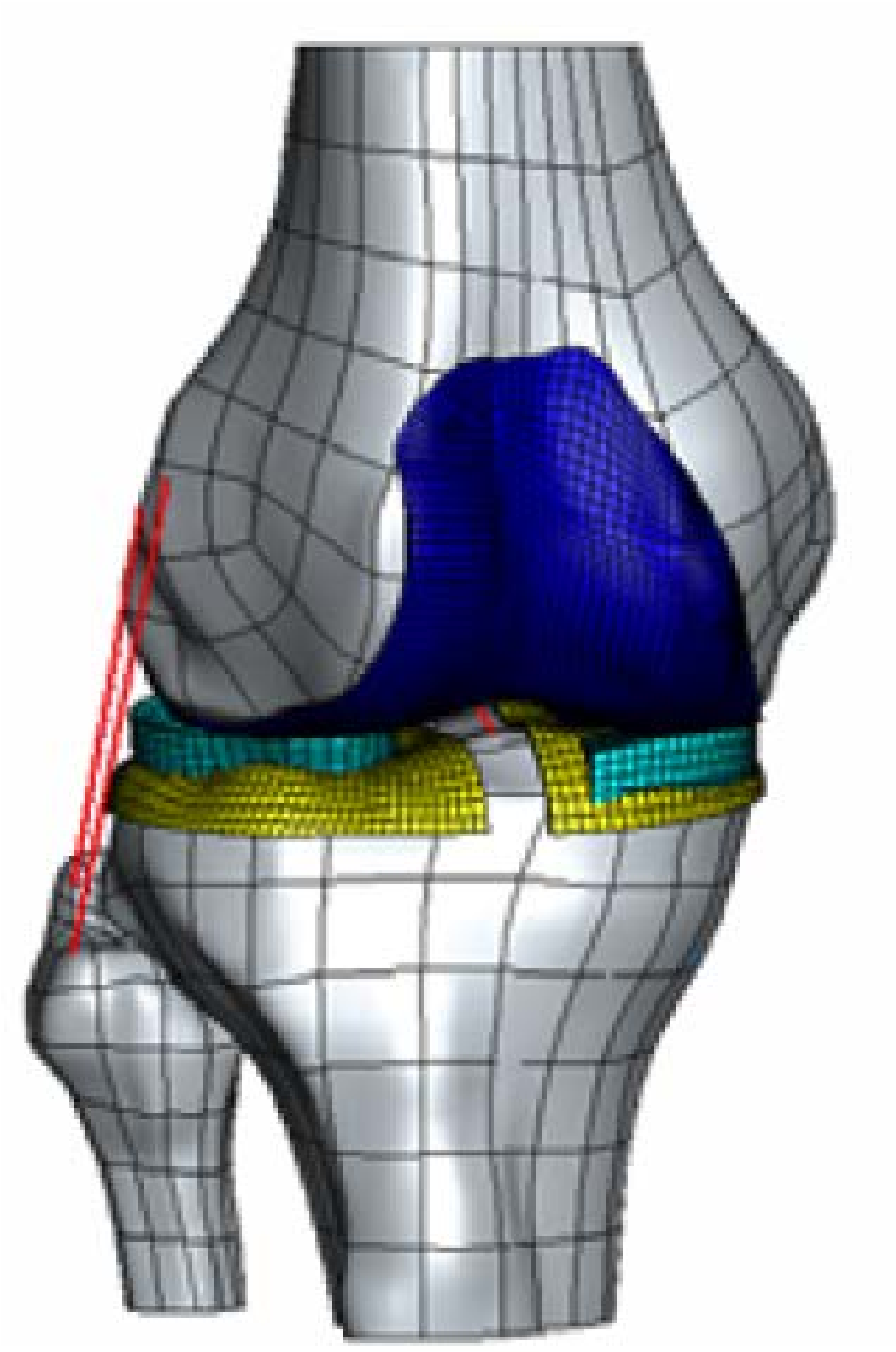


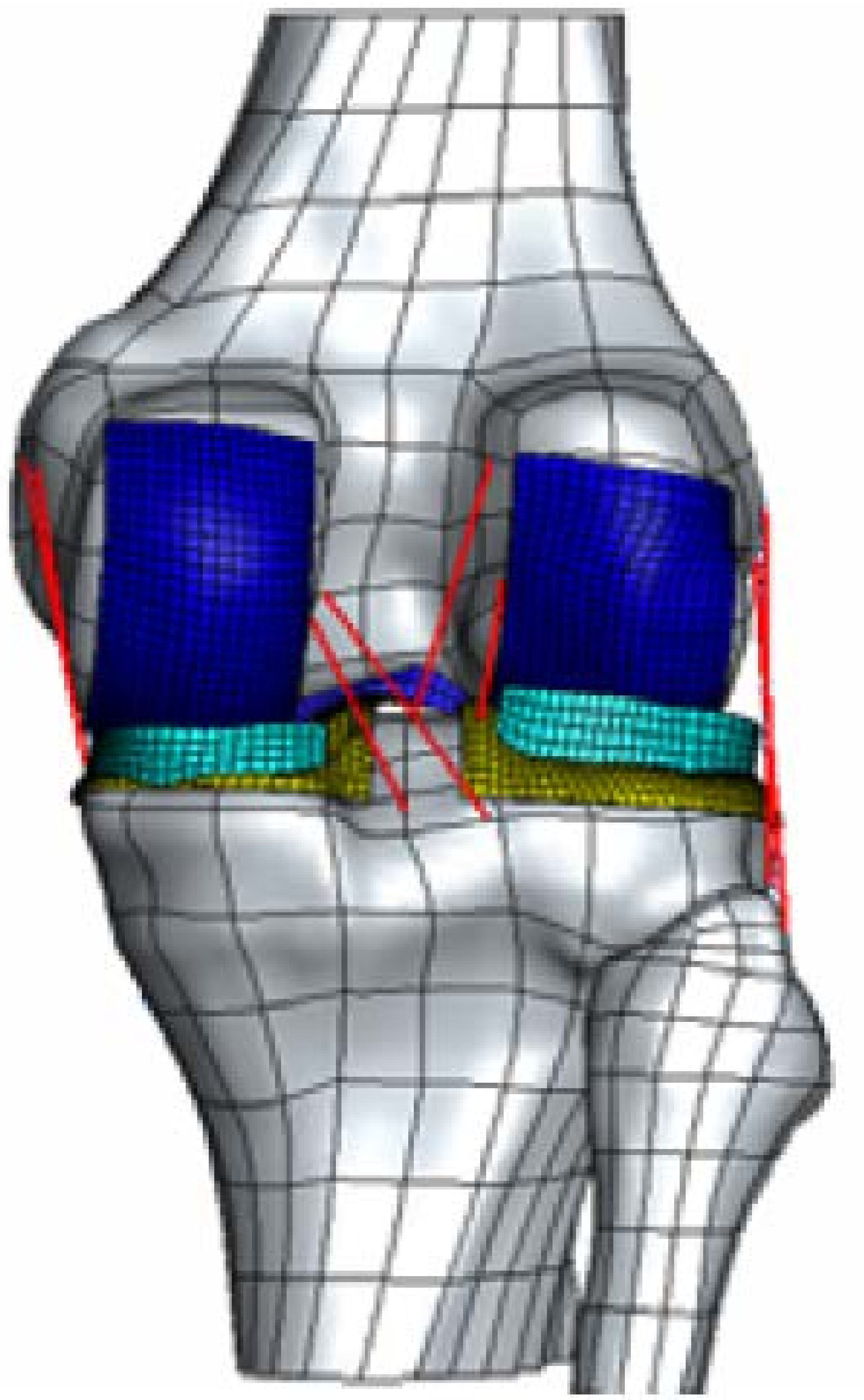
The surgery can be performed in an arthroscopic approach

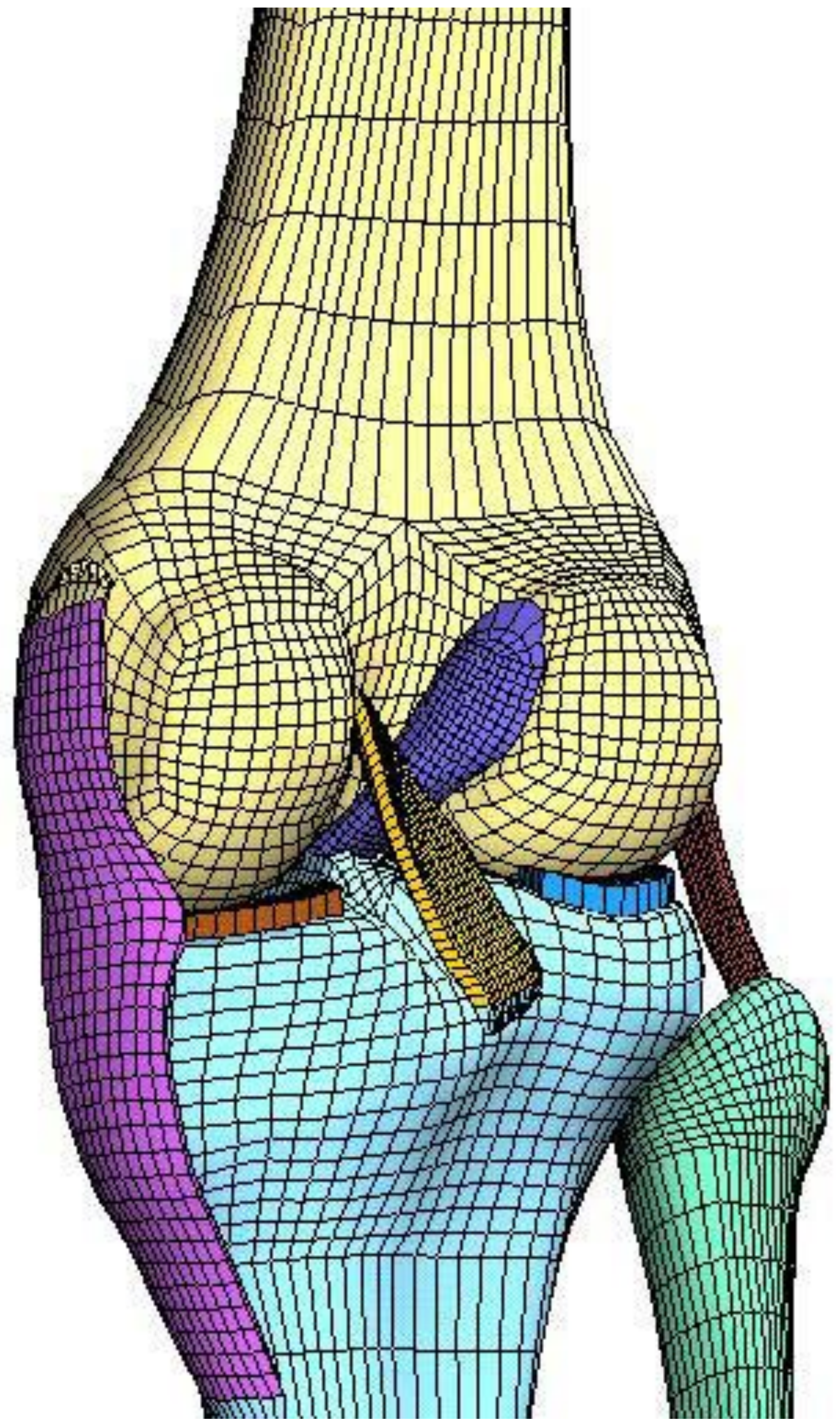


Kato, 2010

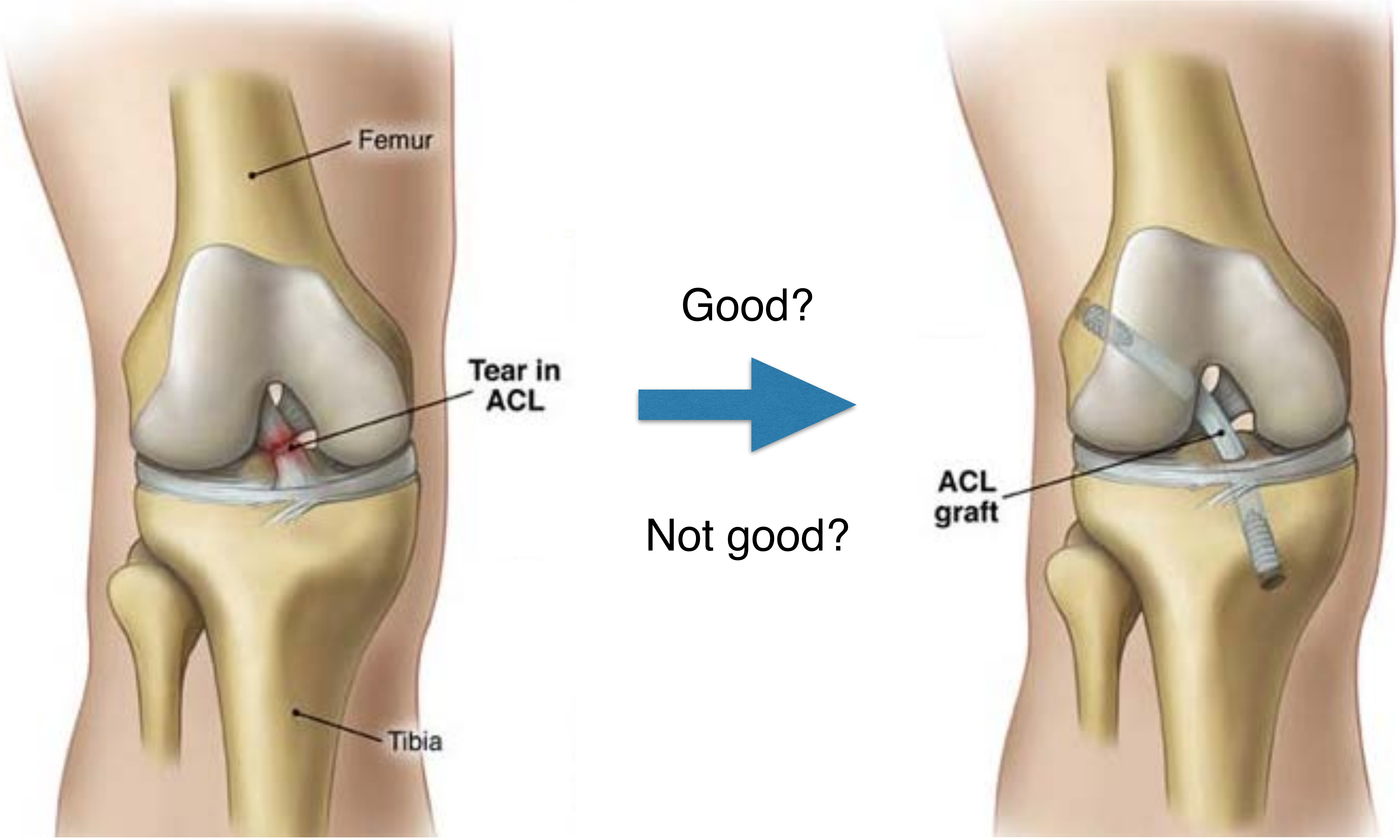
To evaluate the outcome of a complicated biomechanical situation, a numerical analysis is often used







How do we evaluate the success of a ligamentoplasty in general and in particular from a biomechanical point of view?



iHealthSpot, Inc

Finite Element Analysis

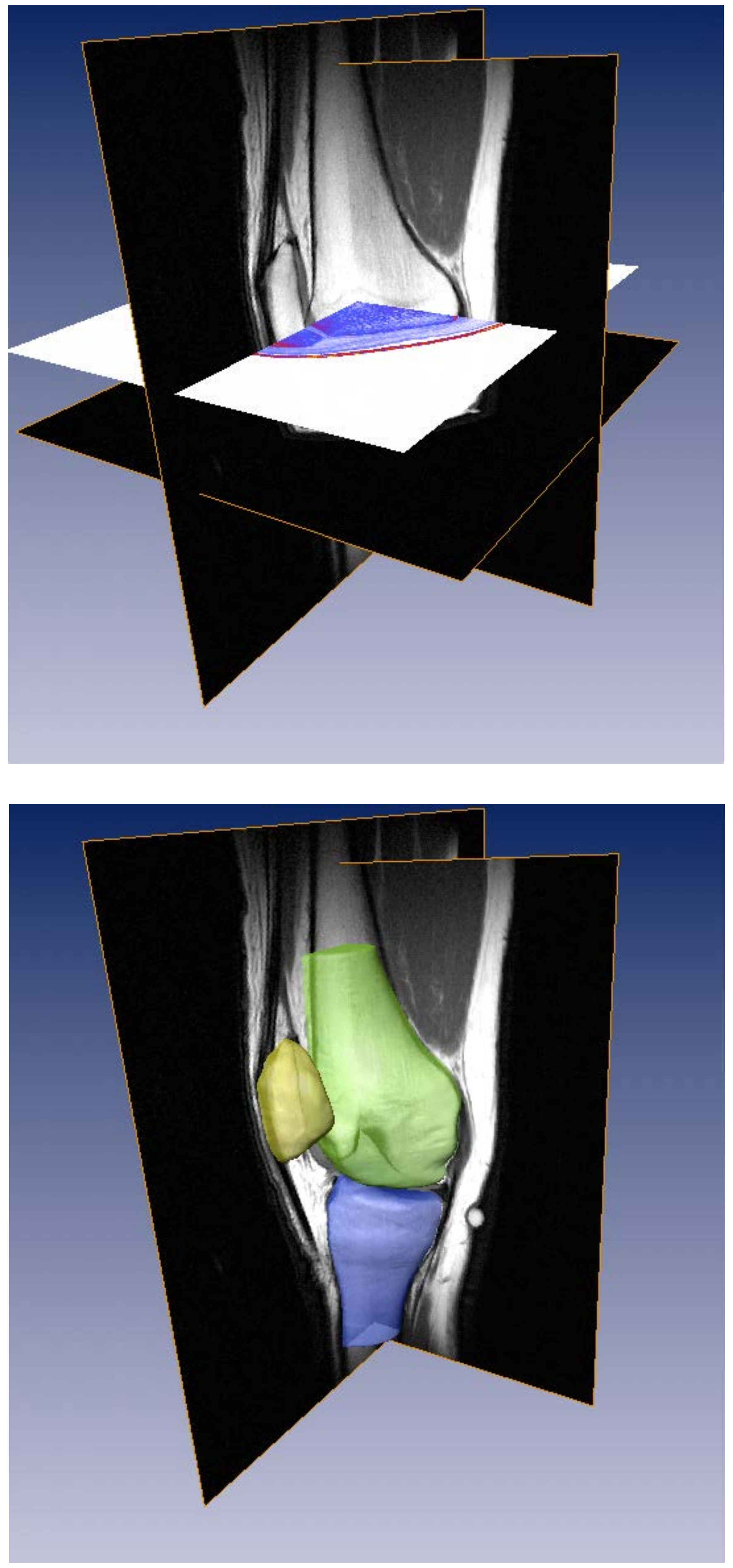
$\boxed{\text{5 steps are necessary}}$

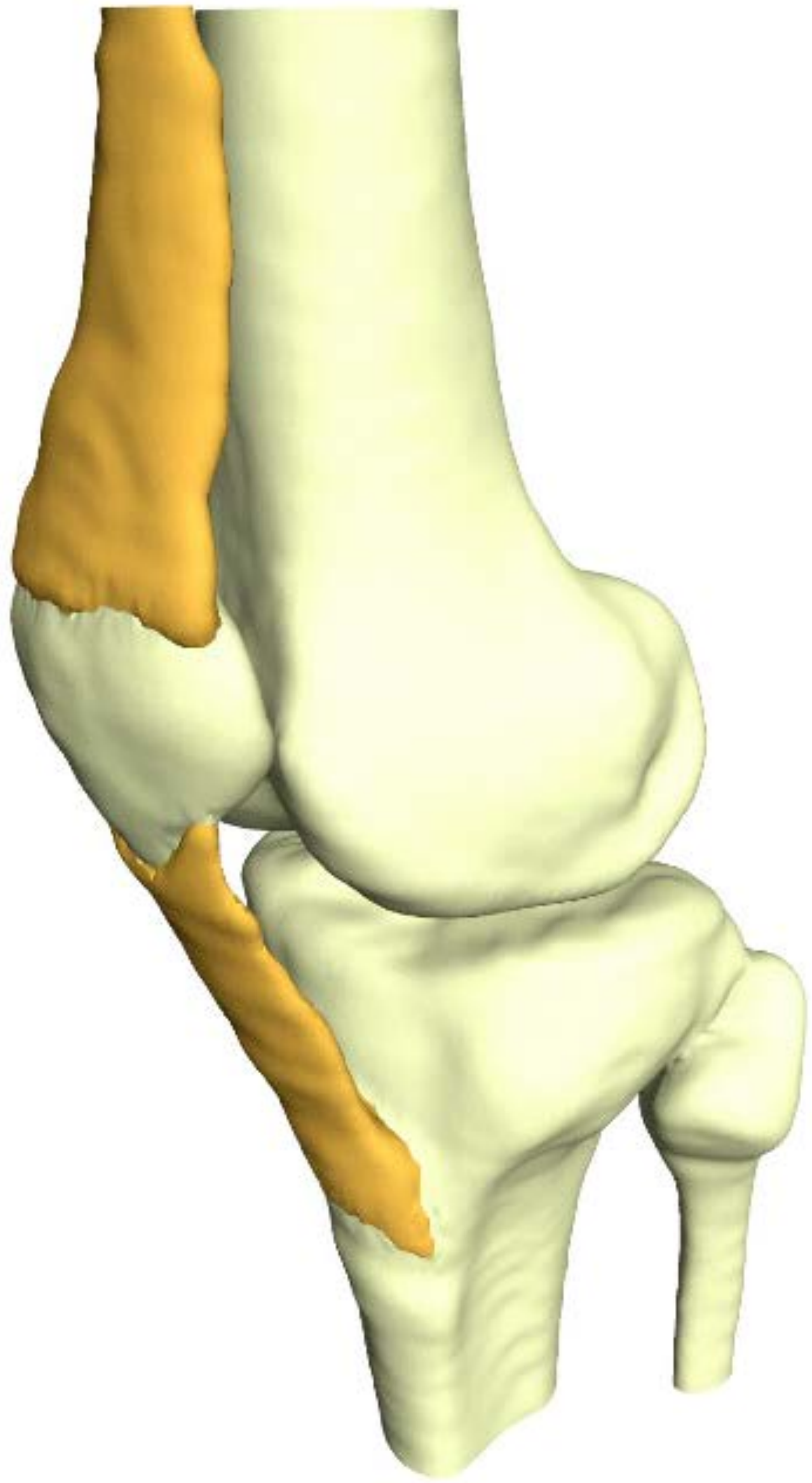
1. Geometry (obtained by MRI or CT)
2. Constitutive laws (mechanical behaviour)
3. Boundary conditions (force or displacement)
4. Meshing
5. Resolution of conservation laws (numerical solver)

-> evaluation of the outcome chosen parameter (in our case, the contact pressure)

Finite Element Analysis

1. Geometry (obtained by MRI and/or CT)





Finite Element Analysis

2. Constitutive laws (mechanical behavior)

Linear

Bone -> linear elastic isotropic -> $\boldsymbol{\sigma} = \lambda(\mathrm{tr}\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}$ -> λ_{bone}, μ_{bone}

Cartilage -> linear elastic isotropic -> $\boldsymbol{\sigma} = \lambda(\mathrm{tr}\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}$ -> λ_{cart}, μ_{cart}

Meniscus -> linear elastic isotropic -> $\boldsymbol{\sigma} = \lambda(\mathrm{tr}\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}$ -> λ_{menis}, μ_{menis}

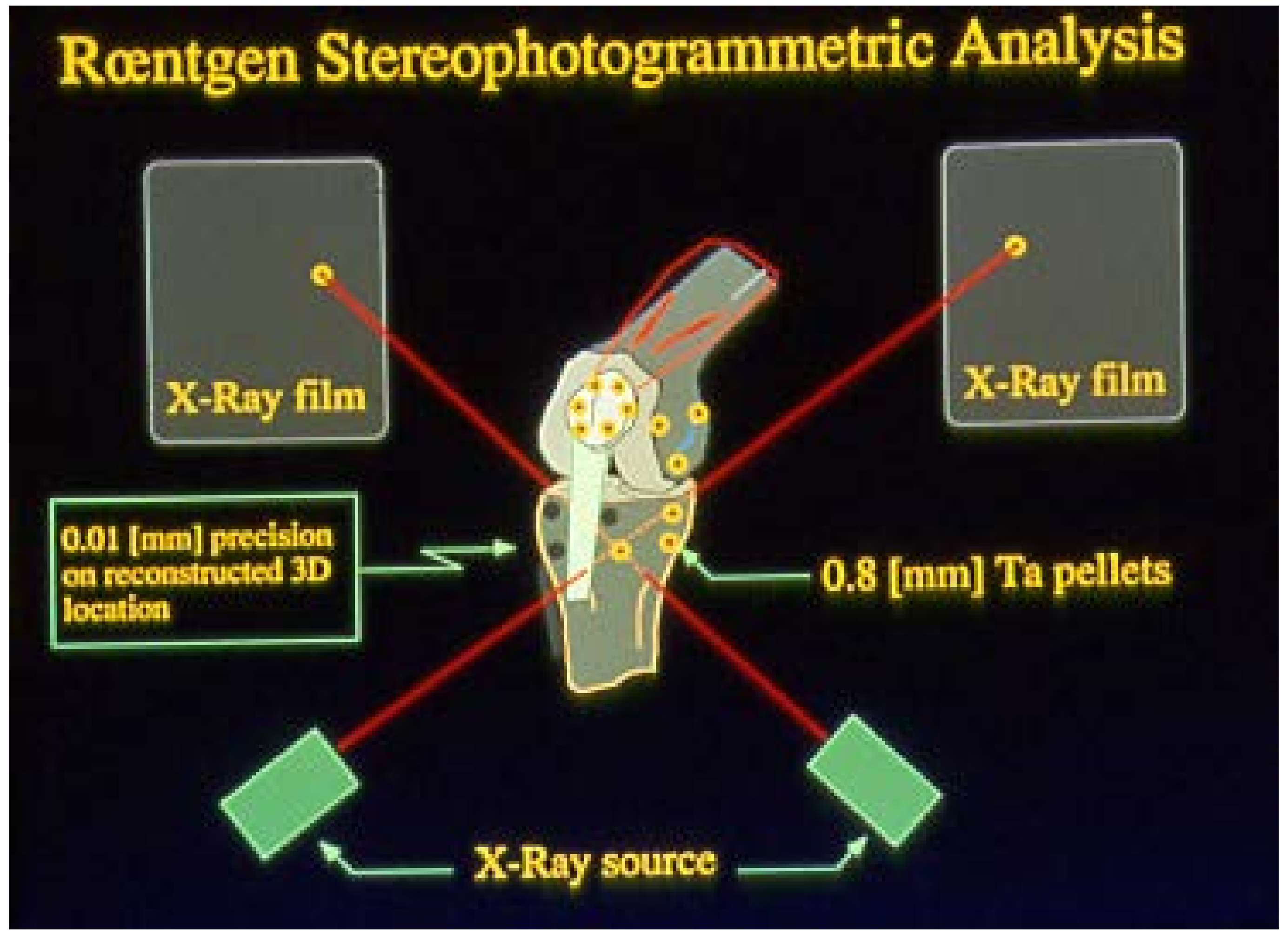
Non-linear

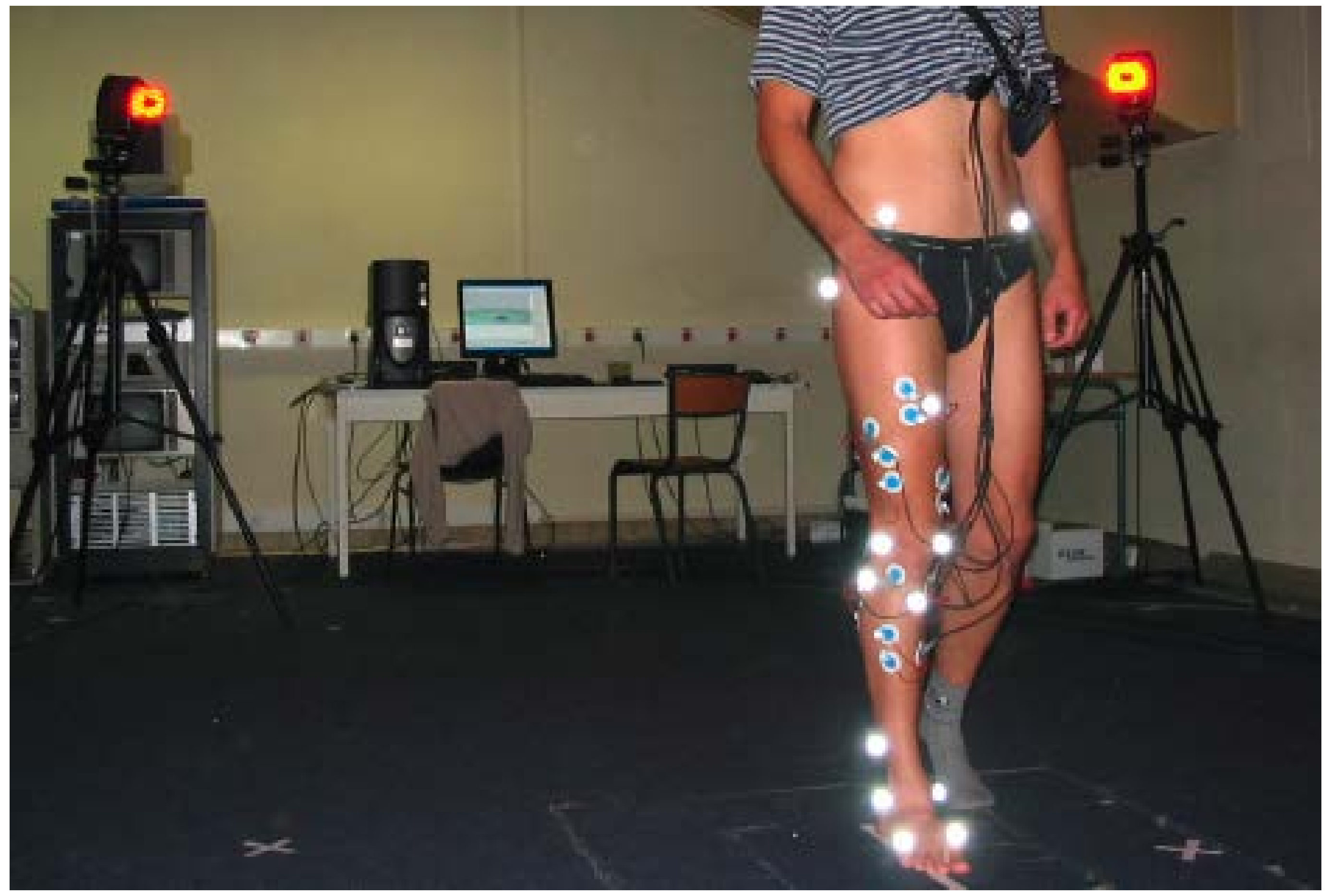
Ligament -> non-linear elastic isotropic -> $\sigma = \alpha(e^{\beta\varepsilon} - 1)$ -> α_{lig}, β_{lig}

Graft -> non-linear elastic isotropic -> $\sigma = \alpha(e^{\beta\varepsilon} - 1)$ -> α_{graft}, β_{graft}

Finite Element Analysis

3. Boundary conditions (force or displacement)



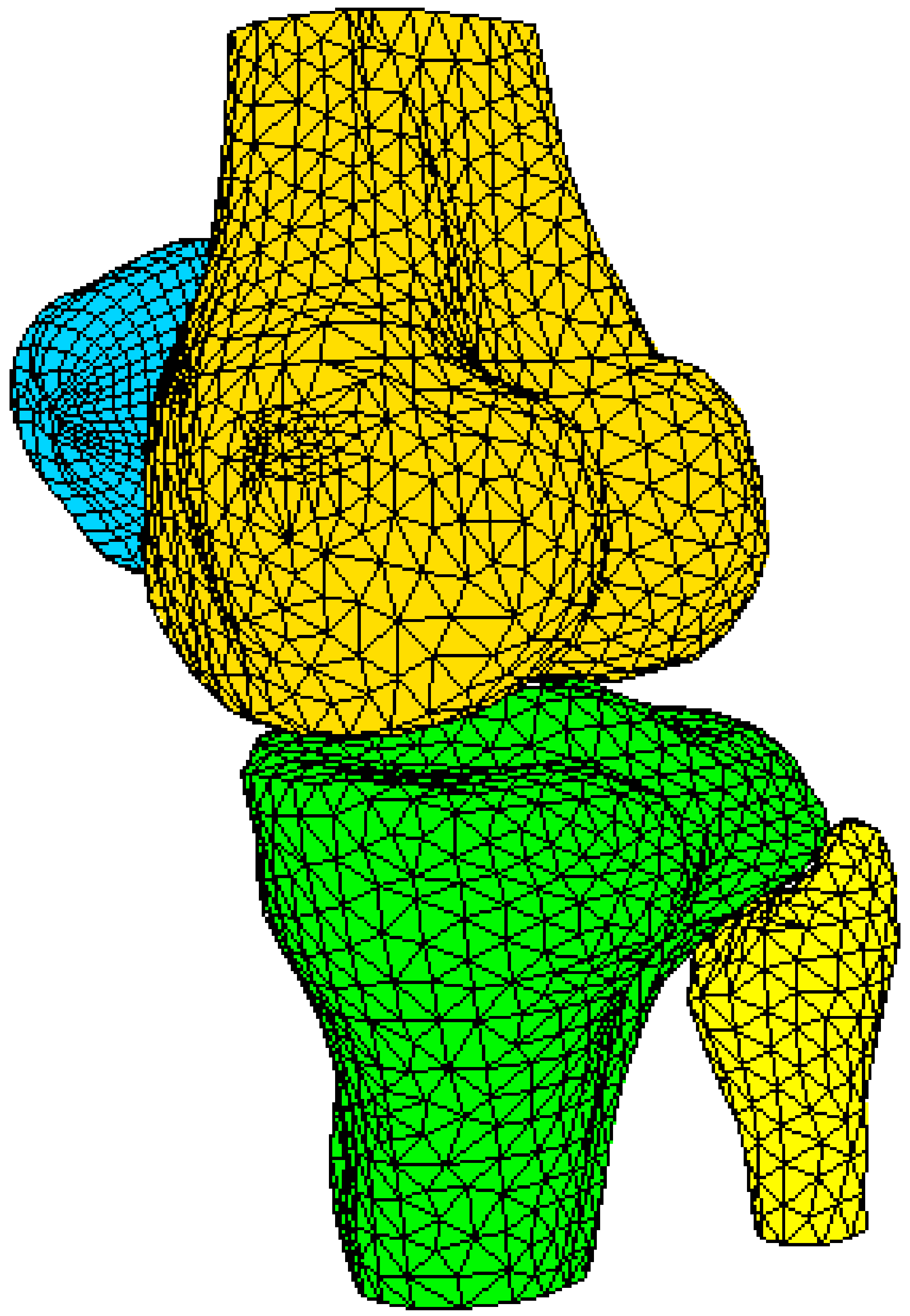


source: Analyse du mouvement, Prof. L. Cheze

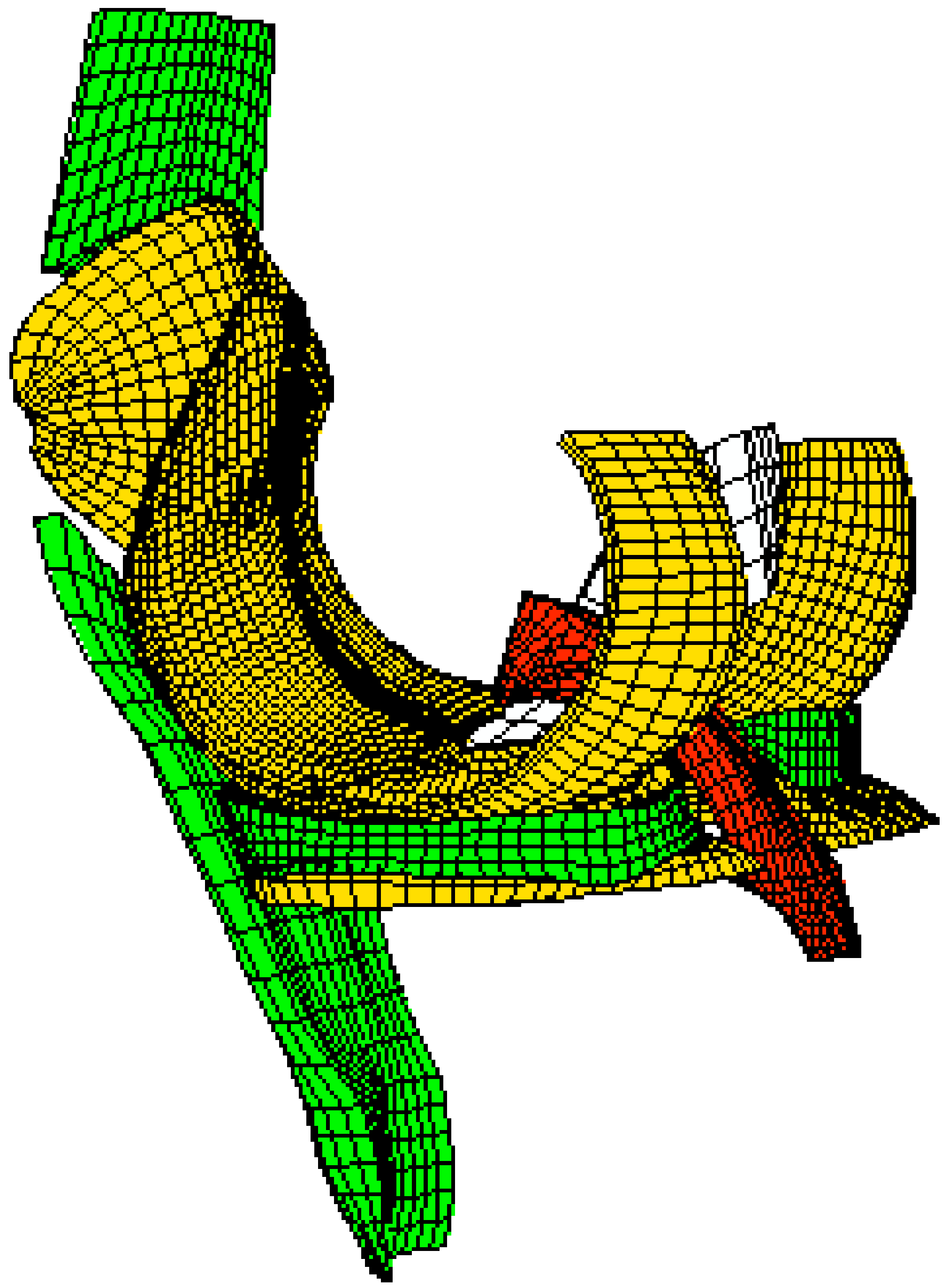
source: Prof.. K. Aminian (LMAM/EPFL)

Finite Element Analysis

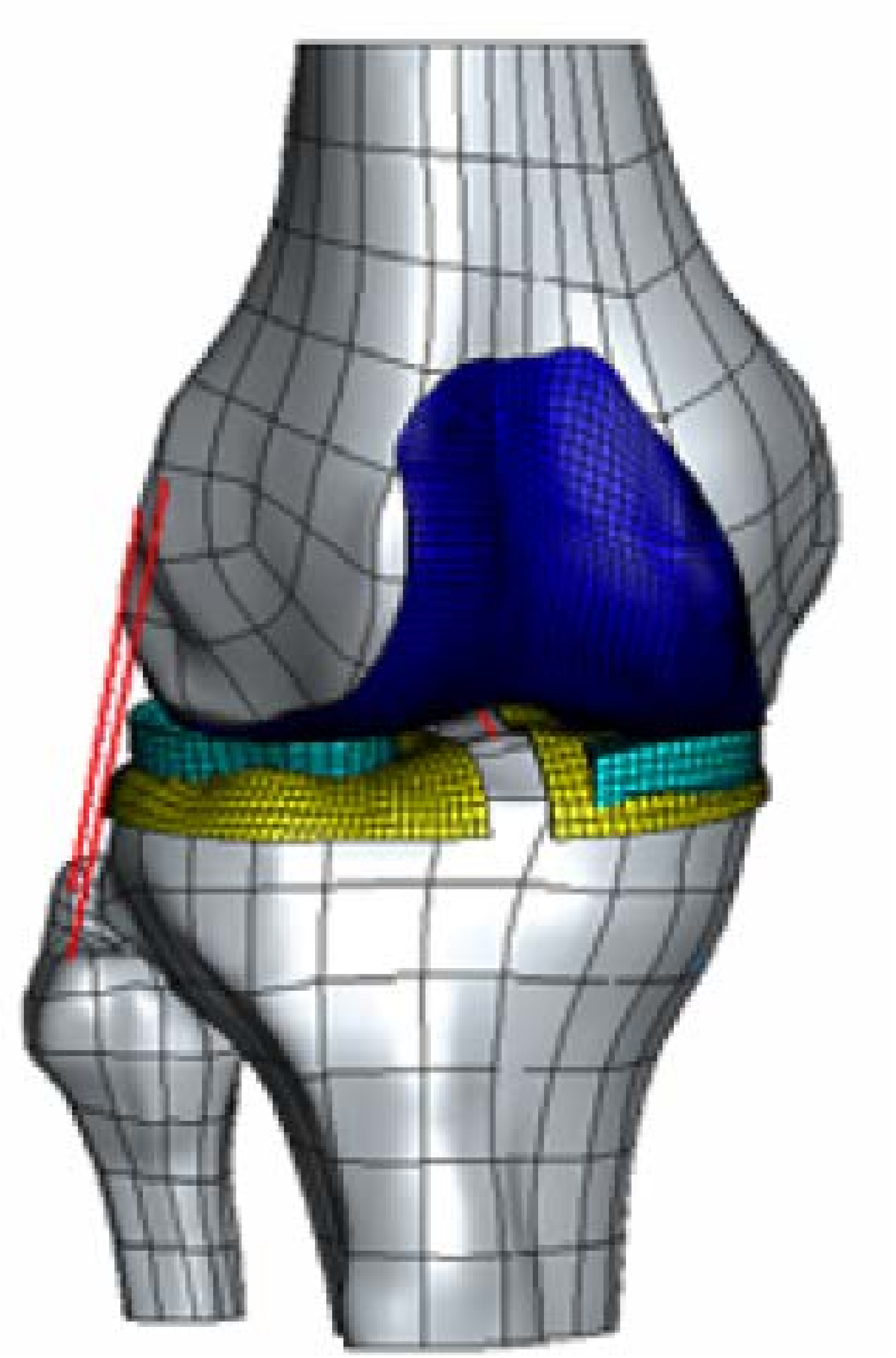
4. Meshing



Hard tissue (bone)



Soft tissues

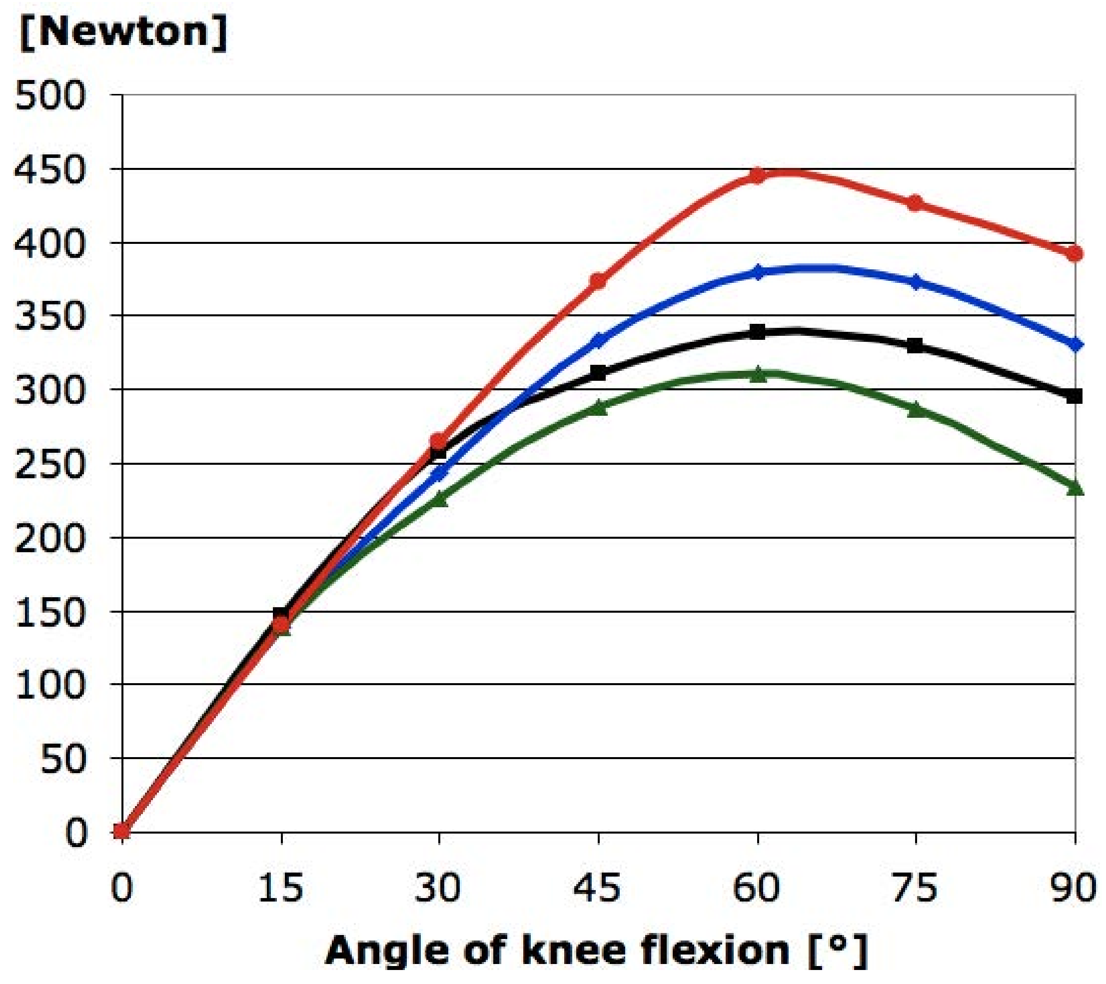


Full model

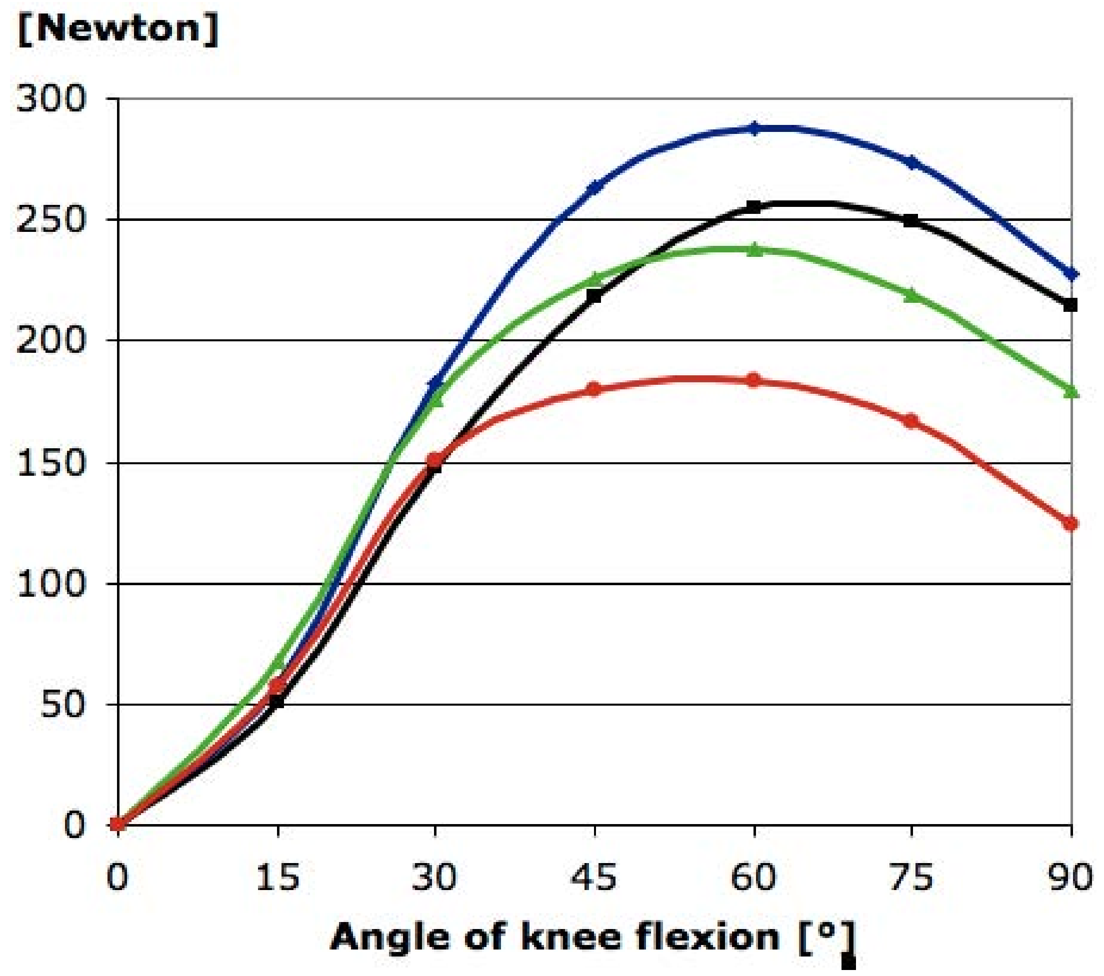
Finite Element Analysis

5. Resolution of conservation laws (numerical solver)

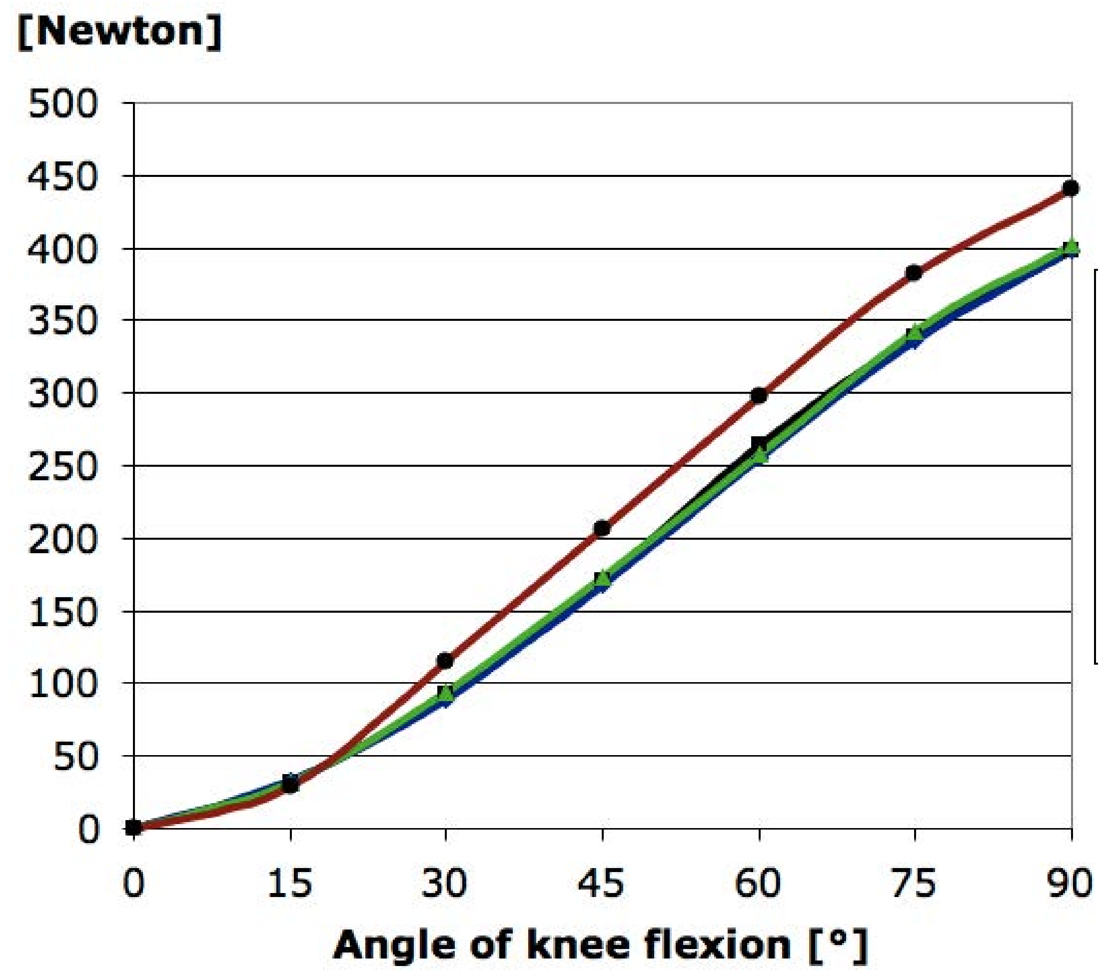
Contact pressure in the different knee compartment



Medial tibiofemoral



Lateral tibiofemoral



Femoropatellar

Native ACL — One bundle graft

No ACL — Double bundle graft

One important aspect of biomechanics is then to characterise tissues through constitutive laws

$$\boxed{\rho \frac{d\boldsymbol{v}}{dt} = div\ \boldsymbol{\sigma} + \rho \boldsymbol{b}}$$

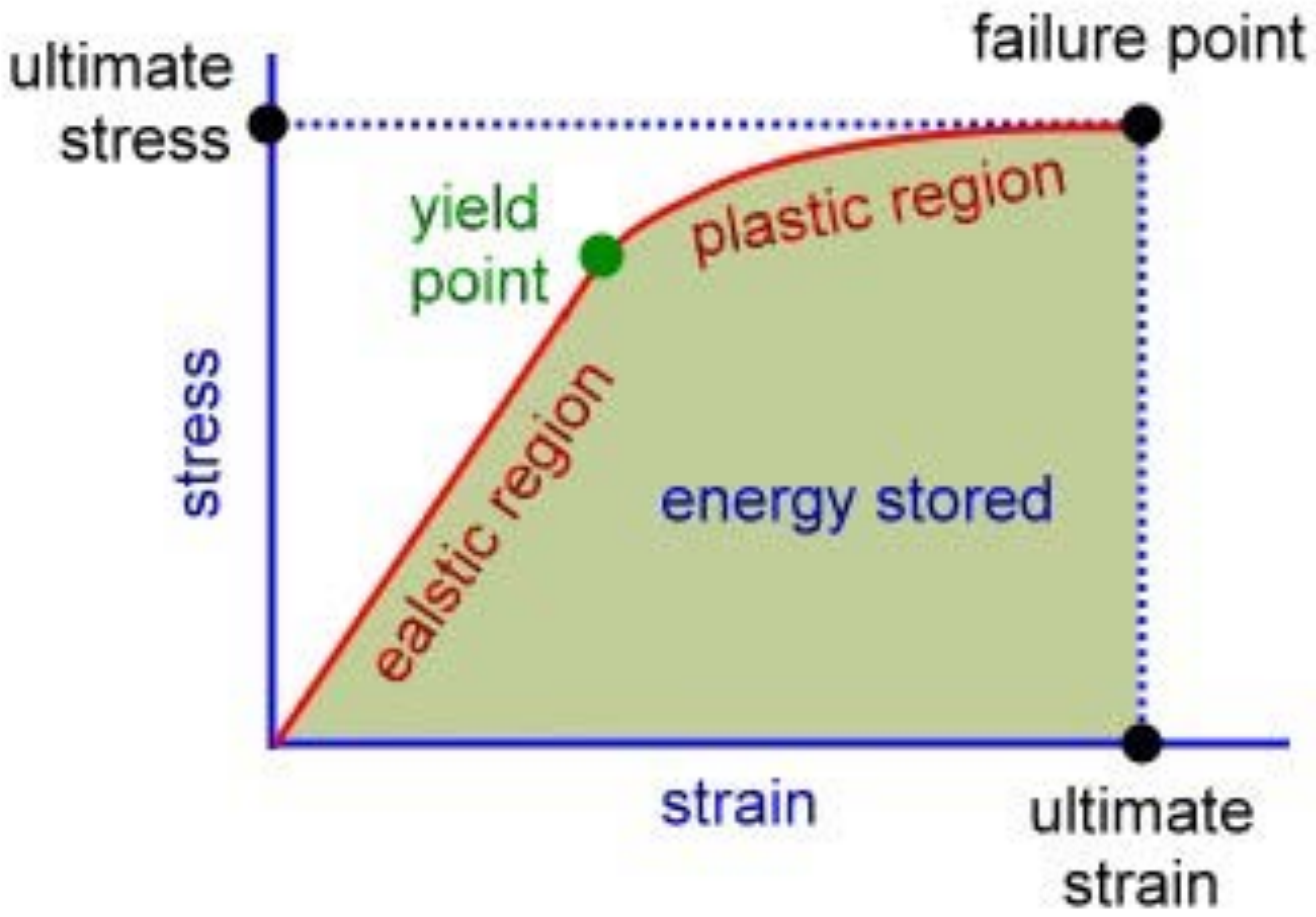
$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}, \varepsilon_p, \dots)$$

Elasticity -> σ = σ(ε) → Linear

→ Non-linear

Elasticity represents only a limited part of the material behavior

- linear elasticity
- non-linear elasticity
- viscoelasticity
- poroelasticity
- poroviscoelasticity
- plasticity
-



Stress-Strain Curve of the Bone